

appendix 1
to chapter

9

Duration Gap Analysis

An alternative method for measuring interest-rate risk, called *duration gap analysis*, examines the sensitivity of the market value of the financial institution's net worth to changes in interest rates. Duration analysis is based on Macaulay's concept of *duration*, which measures the average lifetime of a security's stream of payments (described in the appendix to Chapter 4). Recall that duration is a useful concept, because it provides a good approximation, particularly when interest-rate changes are small, of the sensitivity of a security's market value to a change in its interest rate using the following formula:

$$\% \Delta P \approx -DUR \times \frac{\Delta i}{1 + i} \quad (1)$$

where

$\% \Delta P = (P_{t+1} - P_t)/P_t$ = percent change in market value of the security

DUR = duration

i = interest rate

After having determined the duration of all assets and liabilities on the bank's balance sheet, the bank manager could use this formula to calculate how the market value of each asset and liability changes when there is a change in interest rates and then calculate the effect on net worth. There is, however, an easier way to go about doing this, derived from the basic fact about duration we learned in the appendix to Chapter 4: Duration is additive; that is, the duration of a portfolio of securities is the weighted average of the durations of the individual securities, with the weights reflecting the proportion of the portfolio invested in each. What this means is that the bank manager can figure out the effect that interest-rate changes will have on the market value of net worth by calculating the average duration for assets and for liabilities and then using those figures to estimate the effects of interest-rate changes.

To see how a bank manager would do this, let's return to the balance sheet of the First Bank. The bank manager has already used the procedures outlined in the appendix to Chapter 4 to calculate the duration of each asset and liability, as listed in Table 1. For each asset, the manager then calculates the weighted duration by multiplying the duration times the amount of the asset divided by total assets, which in this case is \$100 million. For example, in the case of securities with maturities less than one year, the manager multiplies the 0.4 year of duration times \$5 million divided by \$100 million to get a weighted duration of 0.02. (Note that physical assets have no cash payments, so they have a duration of zero years.) Doing this for all the assets and

Table 1 Duration of the First Bank's Assets and Liabilities

	Amount (\$ millions)	Duration (years)	Weighted Duration (years)
Assets			
Reserves and cash items	5	0.0	0.00
Securities			
Less than 1 year	5	0.4	0.02
1 to 2 years	5	1.6	0.08
Greater than 2 years	10	7.0	0.70
Residential mortgages			
Variable-rate	10	0.5	0.05
Fixed-rate (30-year)	10	6.0	0.60
Commercial loans			
Less than 1 year	15	0.7	0.11
1 to 2 years	10	1.4	0.14
Greater than 2 years	25	4.0	1.00
Physical capital	5	0.0	<u>0.00</u>
Average duration			2.70
Liabilities			
Checkable deposits	15	2.0	0.32
Money market deposit accounts	5	0.1	0.01
Savings deposits	15	1.0	0.16
CDs			
Variable-rate	10	0.5	0.05
Less than 1 year	15	0.2	0.03
1 to 2 years	5	1.2	0.06
Greater than 2 years	5	2.7	0.14
Overnight funds	5	0.0	0.00
Borrowings			
Less than 1 year	10	0.3	0.03
1 to 2 years	5	1.3	0.07
Greater than 2 years	5	3.1	<u>0.16</u>
Average duration			1.03

adding them up, the bank manager gets a figure for the average duration of the assets of 2.70 years.

The manager follows a similar procedure for the liabilities, noting that total liabilities excluding capital are \$95 million. For example, the weighted duration for checkable deposits is determined by multiplying the 2.0-year duration by \$15 million divided by \$95 million to get 0.32. Adding up these weighted durations, the manager obtains an average duration of liabilities of 1.03 years.

EXAMPLE 1: Duration Gap Analysis

The bank manager wants to know what happens when interest rates rise from 10% to 11%. The total asset value is \$100 million, and the total liability value is \$95 million. Use Equation 1 to calculate the change in the market value of the assets and liabilities.

Solution

With a total asset value of \$100 million, the market value of assets falls by \$2.5 million ($\$100 \text{ million} \times 0.025 = \2.5 million):

$$\% \Delta P = -DUR \times \frac{\Delta i}{1 + i}$$

where

$$\begin{aligned} DUR &= \text{duration} &&= 2.70 \\ \Delta i &= \text{change in interest rate} = 0.11 - 0.10 &&= 0.01 \\ i &= \text{interest rate} &&= 0.10 \end{aligned}$$

Thus:

$$\% \Delta P \approx -2.70 \times \frac{0.01}{1 + 0.10} = -0.025 = -2.5\%$$

With total liabilities of \$95 million, the market value of liabilities falls by \$0.9 million ($\$95 \text{ million} \times 0.009 = -\0.9 million):

$$\% \Delta P \approx -DUR \times \frac{\Delta i}{1 + i}$$

where

$$\begin{aligned} DUR &= \text{duration} &&= 1.03 \\ \Delta i &= \text{change in interest rate} = 0.11 - 0.10 &&= 0.01 \\ i &= \text{interest rate} &&= 0.10 \end{aligned}$$

Thus:

$$\% \Delta P \approx -1.03 \times \frac{0.01}{1 + 0.10} = -0.009 = -0.9\%$$

The result is that the net worth of the bank would decline by \$1.6 million ($-\$2.5 \text{ million} - (-\$0.9 \text{ million}) = -\$2.5 \text{ million} + \$0.9 \text{ million} = -\1.6 million).

The bank manager could have gotten to the answer even more quickly by calculating what is called a *duration gap*, which is defined as follows:

$$DUR_{gap} = DUR_a - \left(\frac{L}{A} \times DUR_l \right) \quad (2)$$

where DUR_a = average duration of assets

DUR_l = average duration of liabilities

L = market value of liabilities

A = market value of assets

EXAMPLE 2: Duration Gap Analysis

Based on the information provided in Example 1, use Equation 2 to determine the duration gap for First Bank.

Solution

The duration gap for First Bank is 1.72 years:

$$DUR_{gap} = DUR_a - \left(\frac{L}{A} \times DUR_l \right)$$

where

$$\begin{aligned} DUR_a &= \text{average duration of assets} &&= 2.70 \\ L &= \text{market value of liabilities} &&= 95 \\ A &= \text{market value of assets} &&= 100 \\ DUR_l &= \text{average duration of liabilities} &&= 1.03 \end{aligned}$$

Thus:

$$DUR_{gap} = 2.70 - \left(\frac{95}{100} \times 1.03 \right) = 1.72 \text{ years}$$

To estimate what will happen if interest rates change, the bank manager uses the DUR_{gap} calculation in Equation 1 to obtain the change in the market value of net worth as a percentage of total assets. In other words, the change in the market value of net worth as a percentage of assets is calculated as:

$$\frac{\Delta NW}{A} \approx -DUR_{gap} \times \frac{\Delta i}{1 + i} \quad (3)$$

EXAMPLE 3: Duration Gap Analysis

What is the change in the market value of net worth as a percentage of assets if interest rates rise from 10% to 11%? (Use Equation 3.)

Solution

A rise in interest rates from 10% to 11% would lead to a change in the market value of net worth as a percentage of assets of -1.6% :

$$\frac{\Delta NW}{A} = -DUR_{gap} \times \frac{\Delta i}{1 + i}$$

where

$$\begin{aligned} DUR_{gap} &= \text{duration gap} &&= 1.72 \\ \Delta i &= \text{change in interest rate} = 0.11 - 0.10 &&= 0.01 \\ i &= \text{interest rate} &&= 0.10 \end{aligned}$$

Thus:

$$\frac{\Delta NW}{A} = -1.72 \times \frac{0.01}{1 + 0.10} = -0.016 = -1.6\%$$

With assets totaling \$100 million, Example 3 indicates a fall in the market value of net worth of \$1.6 million, which is the same figure that we found in Example 1.

As our examples make clear, both income gap analysis and duration gap analysis indicate that the First Bank will suffer from a rise in interest rates. Indeed, in this example, we have seen that a rise in interest rates from 10% to 11% will cause the market value of net worth to fall by \$1.6 million, which is one-third the initial amount of bank capital. Thus the bank manager realizes that the bank faces substantial interest-rate risk because a rise in interest rates could cause it to lose a lot of its capital. Clearly, income gap analysis and duration gap analysis are useful tools for telling a financial institution manager the institution's degree of exposure to interest-rate risk.

Study Guide

To make sure that you understand income gap and duration gap analysis, you should be able to verify that if interest rates fall from 10% to 5%, the First Bank will find its income increasing and the market value of its net worth rising.

Example of a Nonbanking Financial Institution

So far we have focused on an example involving a banking institution that has borrowed short and lent long so that when interest rates rise, both income and the net worth of the institution fall. It is important to recognize that income and duration gap analysis applies equally to other financial institutions. Furthermore, it is important for you to see that some financial institutions have income and duration gaps that are opposite in sign to those of banks, so that when interest rates rise, both income and net worth rise rather than fall. To get a more complete picture of income and duration gap analysis, let us look at a nonbank financial institution, the Friendly Finance Company, which specializes in making consumer loans.

The Friendly Finance Company has the following balance sheet:

Friendly Finance Company			
Assets		Liabilities	
Cash and deposits	\$3 million	Commercial paper	\$40 million
Securities		Bank loans	
Less than 1 year	\$5 million	Less than 1 year	\$3 million
1 to 2 years	\$1 million	1 to 2 years	\$2 million
Greater than 2 years	\$1 million	Greater than 2 years	\$5 million
Consumer loans		Long-term bonds and	
Less than 1 year	\$50 million	other long-term debt	\$40 million
1 to 2 years	\$20 million	Capital	\$10 million
Greater than 2 years	\$15 million		
Physical capital	\$5 million		
Total	\$100 million	Total	\$100 million

The manager of the Friendly Finance Company calculates the rate-sensitive assets to be equal to the \$5 million of securities with maturities less than one year plus the \$50 million of consumer loans with maturities of less than one year, for a total of \$55

million of rate-sensitive assets. The manager then calculates the rate-sensitive liabilities to be equal to the \$40 million of commercial paper, all of which has a maturity of less than one year, plus the \$3 million of bank loans maturing in less than a year, for a total of \$43 million. The calculation of the income gap is then:

$$GAP = RSA - RSL = \$55 \text{ million} - \$43 \text{ million} = \$12 \text{ million}$$

To calculate the effect on income if interest rates rise by 1%, the manager multiplies the *GAP* of \$12 million times the change in the interest rate to get the following:

$$\Delta I = GAP \times \Delta i = \$12 \text{ million} \times 1\% = \$120,000$$

Thus the manager finds that the finance company's income will rise by \$120,000 when interest rates rise by 1%. The reason that the company has benefited from the interest-rate rise, in contrast to the First Bank, whose profits suffer from the rise in interest rates, is that the Friendly Finance Company has a positive income gap because it has more rate-sensitive assets than liabilities.

Like the bank manager, the manager of the Friendly Finance Company is also interested in what happens to the market value of the net worth of the company when interest rates rise by 1%. So the manager calculates the weighted duration of each item in the balance sheet, adds them up as in Table 2, and obtains a duration for the assets of 1.16 years and for the liabilities, 2.77 years. The duration gap is then calculated to be:

$$DUR_{gap} = DUR_a - \left(\frac{L}{A} \times DUR_l \right) = 1.16 - \left(\frac{90}{100} \times 2.77 \right) = -1.33 \text{ years}$$

Since the Friendly Finance Company has a negative duration gap, the manager realizes that a rise in interest rates by 1 percentage point from 10% to 11% will increase the market value of net worth of the firm. The manager checks this by calculating the change in the market value of net worth as a percentage of assets:

$$\Delta NW = -DUR_{gap} \times \frac{\Delta i}{1 + i} = -(-1.33) \times \frac{0.01}{1 + 0.10} = 0.012 = 1.2\%$$

With assets of \$100 million, this calculation indicates that net worth will rise in market value by \$1.2 million.

Even though the income and duration gap analysis indicates that the Friendly Finance Company gains from a rise in interest rates, the manager realizes that if interest rates go in the other direction, the company will suffer a fall in income and market value of net worth. Thus the finance company manager, like the bank manager, realizes that the institution is subject to substantial interest-rate risk.

Some Problems with Income and Duration Gap Analysis

Although you might think that income and duration gap analysis is complicated enough, further complications make a financial institution manager's job even harder.

One assumption that we have been using in our discussion of income and duration gap analysis is that when the level of interest rates changes, interest rates on all maturities change by exactly the same amount. That is the same as saying that we conducted our analysis under the assumption that the slope of the yield curve remains unchanged. Indeed, the situation is even worse for duration gap analysis, because the duration gap is calculated assuming that interest rates for all maturities are the same—in other words, the yield curve is assumed to be flat. As our discussion of the term

Table 2 Duration of the Friendly Finance Company's Assets and Liabilities

	Amount (\$ millions)	Duration (years)	Weighted Duration (years)
Assets			
Cash and deposits	3	0.0	0.00
Securities			
Less than 1 year	5	0.5	0.05
1 to 2 years	1	1.7	0.02
Greater than 2 years	1	9.0	0.09
Consumer loans			
Less than 1 year	50	0.5	0.25
1 to 2 years	20	1.5	0.30
Greater than 2 years	15	3.0	0.45
Physical capital	5	0.0	<u>0.00</u>
Average duration			1.16
Liabilities			
Commercial paper	40	0.2	0.09
Bank loans			
Less than 1 year	3	0.3	0.01
1 to 2 years	2	1.6	0.04
Greater than 2 years	5	3.5	0.19
Long-term bonds and other			
long-term debt	40	5.5	<u>2.44</u>
Average duration			2.77

structure of interest rates in Chapter 6 indicated, however, the yield curve is not flat, and the slope of the yield curve fluctuates and has a tendency to change when the level of the interest rate changes. Thus to get a truly accurate assessment of interest-rate risk, a financial institution manager has to assess what might happen to the slope of the yield curve when the level of the interest rate changes and then take this information into account when assessing interest-rate risk. In addition, duration gap analysis is based on the approximation in Equation 1 and thus only works well for small changes in interest rates.

A problem with income gap analysis is that, as we have seen, the financial institution manager must make estimates of the proportion of supposedly fixed-rate assets and liabilities that may be rate-sensitive. This involves estimates of the likelihood of prepayment of loans or customer shifts out of deposits when interest rates change. Such guesses are not easy to make, and as a result, the financial institution manager's estimates of income gaps may not be very accurate. A similar problem occurs in calculating durations of assets and liabilities, because many of the cash payments are uncertain. Thus the estimate of the duration gap might not be accurate either.

Do these problems mean that managers of banks and other financial institutions should give up on gap analysis as a tool for measuring interest-rate risk? Financial institutions do use more sophisticated approaches to measuring interest-rate risk, such as scenario analysis and value-at-risk analysis, which make greater use of computers to more accurately measure changes in prices of assets when interest rates change. Income and duration gap analyses, however, still provide simple frameworks to help financial institution managers to get a first assessment of interest-rate risk, and they are thus useful tools in the financial institution manager's toolkit.

Application

Strategies for Managing Interest-Rate Risk

Once financial institution managers have done the duration and income gap analysis for their institutions, they must decide which alternative strategies to pursue. If the manager of the First Bank firmly believes that interest rates will fall in the future, he or she may be willing to take no action knowing that the bank has more rate-sensitive liabilities than rate-sensitive assets, and so will benefit from the expected interest-rate decline. However, the bank manager also realizes that the First Bank is subject to substantial interest-rate risk, because there is always a possibility that interest rates will rise rather than fall, and as we have seen, this outcome could bankrupt the bank. The manager might try to shorten the duration of the bank's assets to increase their rate sensitivity either by purchasing assets of shorter maturity or by converting fixed-rate loans into adjustable-rate loans. Alternatively, the bank manager could lengthen the duration of the liabilities. With these adjustments to the bank's assets and liabilities, the bank would be less affected by interest-rate swings.

For example, the bank manager might decide to eliminate the income gap by increasing the amount of rate-sensitive assets to \$49.5 million to equal the \$49.5 million of rate-sensitive liabilities. Or the manager could reduce rate-sensitive liabilities to \$32 million so that they equal rate-sensitive assets. In either case, the income gap would now be zero, so a change in interest rates would have no effect on bank profits in the coming year.

Alternatively, the bank manager might decide to immunize the market value of the bank's net worth completely from interest-rate risk by adjusting assets and liabilities so that the duration gap is equal to zero. To do this, the manager can set DUR_{gap} equal to zero in Equation 2 and solve for DUR_a :

$$DUR_a = \frac{L}{A} \times DUR_l = \frac{95}{100} \times 1.03 = 0.98$$

These calculations reveal that the manager should reduce the average duration of the bank's assets to 0.98 year. To check that the duration gap is set equal to zero, the calculation is:

$$DUR_{gap} = 0.98 - \left(\frac{95}{100} \times 1.03 \right) = 0$$

In this case, as in Equation 3, the market value of net worth would remain unchanged when interest rates change. Alternatively, the bank manager could calculate the value of the duration of the liabilities that would produce a duration gap of zero. To do this would involve setting DUR_{gap} equal to zero in Equation 2 and solving for DUR_l :

$$DUR_l = DUR_a \times \frac{A}{L} = 2.70 \times \frac{100}{95} = 2.84$$

This calculation reveals that the interest-rate risk could also be eliminated by increasing the average duration of the bank's liabilities to 2.84 years. The manager again checks that the duration gap is set equal to zero by calculating:

$$DUR_{gap} = 2.70 - \left(\frac{95}{100} \times 2.84 \right) = 0$$

Study Guide

To see if you understand how a financial institution manager can protect income and net worth from interest-rate risk, first calculate how the Friendly Finance Company might change the amount of its rate-sensitive assets or its rate-sensitive liabilities to eliminate the income gap. You should find that the income gap can be eliminated either by reducing the amount of rate-sensitive assets to \$43 million or by raising the amount of rate-sensitive liabilities to \$55 million. Also do the calculations to determine what modifications to the duration of the assets or liabilities would immunize the market value of Friendly Finance's net worth from interest-rate risk. You should find that interest-rate risk would be eliminated if the duration of the assets were set to 2.49 years or if the duration of the liabilities were set to 1.29 years.

One problem with eliminating a financial institution's interest-rate risk by altering the balance sheet is that doing so might be very costly in the short run. The financial institution may be locked into assets and liabilities of particular durations because of its field of expertise. Fortunately, recently developed financial instruments, such as financial futures, options, and interest-rate swaps, help financial institutions manage their interest-rate risk without requiring them to rearrange their balance sheets. We discuss these instruments and how they can be used to manage interest-rate risk in Chapter 13.