3

Resistance

Outline

Learning Outcomes

After completing this chapter you will be able to

- explain the nature of resistance
- describe the relationship between resistance and conductance
- determine the resistance of conductors with various cross-sectional areas and with different lengths
- explain the effect of temperature on resistance and solve problems involving temperature change
- explain the difference between fixed resistors and variable resistors
- determine the size and tolerance of commercial resistors using the resistor colour code
- describe the characteristics of various types of resistive devices, such as thermistors, varistors, and photoconductive cells

Key Terms

3.1 Introduction

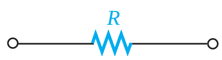


FIG. 3.1
Resistance symbol and notation.

The flow of charge through any material encounters an opposing force similar in many respects to mechanical friction. The collision of electrons with the ions of the material converts electrical energy to heat. The opposition to the current is called **resistance**. The unit of measurement of resistance is the **ohm**, for which the symbol is Ω, the capital Greek letter *omega*. The circuit symbol for resistance appears in Fig. 3.1 with the graphic abbreviation for resistance (R).

The resistance of any material with a uniform cross-sectional area is determined by the following four factors:

1. ***Type of material***
2. ***Length***
3. ***Cross-sectional area***
4. ***Temperature***

Each material, with its unique molecular structure, will react differently to pressures to establish current flow. Conductors that permit a generous flow of charge with little external pressure will have low resistance, while insulators will have high resistance.

As one might expect, the longer the path the charge must pass through, the higher the resistance, and the larger the area, the lower the resistance. Resistance is thus directly proportional to length and inversely proportional to area.

For most conductors, as the temperature increases, the increased motion of the ions within the molecular structure makes it increasingly difficult for the free electrons to pass through, and the resistance increases.

At a fixed temperature of 20°C (room temperature), the resistance is related to the other three factors by

$$\boxed{R = \rho \frac{l}{A}} \qquad \text{(ohms, } \Omega) \qquad (3.1)$$

where ρ (Greek letter *rho*) is a characteristic of the material called the **resistivity**, l is the length of the sample, and A is the cross-sectional area of the sample.

The units of measurement for Eq. (3.1) depend on the application. For most applications (e.g., integrated circuits) the units are as defined in Section 3.2.

3.2 Resistance: Metric Units

The design of resistive elements for various areas of application, including circular conductors, thin-film resistors and integrated circuits, uses metric units for the quantities of Eq. (3.1). In SI units, the resistivity would be measured in ohm-metres, the area in square metres, and the length in metres. However, the metre is generally too large a unit of measure for most applications, and so the centimetre is usually used. The resulting dimensions for Eq. (3.1) are therefore

ρ, ohm-centimetres
l, centimetres
A, square centimetres

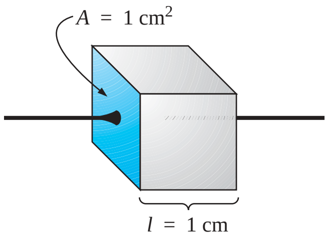


FIG. 3.2
Defining ρ in ohm-centimetres.

TABLE 3.1
Resistivity (r) of various materials in ohm-centimetres.

Material	ρ @ 20°C
Silver	1.645×10^{-6}
Copper	$\mathbf{1.723 \times 10^{-6}}$
Gold	2.443×10^{-6}
Aluminum	2.825×10^{-6}
Tungsten	5.485×10^{-6}
Nickel	7.811×10^{-6}
Iron	12.299×10^{-6}
Tantalum	15.54×10^{-6}
Nichrome	99.72×10^{-6}
Tin oxide	250×10^{-6}
Carbon	3500×10^{-6}

The units for ρ can be derived from

$$\rho = \frac{RA}{l} = \frac{\Omega\cdot\text{cm}^2}{\text{cm}} = \Omega\cdot\text{cm}$$

The resistivity of a material is actually the resistance of a sample such as the one appearing in Fig. 3.2. Table 3.1 provides a list of values of ρ in ohm-centimetres. Note that the area is expressed in square centimetres. For circular wires, the area is related to diameter d by $A = \pi d^2/4$.

EXAMPLE 3.1

What is the resistance of a 100-m length of #22 AWG copper conductor?

Solution:

$$\rho = 1.723 \times 10^{-6}\ \Omega\cdot\text{cm}$$

From Table 3.3, the cross-sectional area is 0.0033 cm^2.
From Equation 3.1:

$$R = \rho\frac{l}{A}$$

$$= \frac{(1.723 \times 10^{-6}\Omega\cdot\text{cm})\ (10\,000\ \text{cm})}{0.0033\text{cm}^2}$$

$$= 5.2\ \Omega$$

EXAMPLE 3.2

An undetermined number of metres have been used from a carton of #14 AWG wire. If the measured resistance is 1.6 Ω, determine the length of wire remaining in the carton.

Solution:

$$\rho = 1.723 \times 10^{-6}\ \Omega\cdot\text{cm}$$

From Table 3.3, the cross-sectional area is 0.0211 cm^2.
From Equation 3.1:

$$R = \rho\frac{l}{A}$$

$$l = \frac{RA}{\rho}$$

$$= \frac{(1.6\ \Omega)\ (0.0211\ \text{cm}^2)}{1.723 \times 10^{-6}\ \Omega\cdot\text{cm}}$$

$$= 19\,594\ \text{cm}$$

$$= 196\ \text{m}$$

The resistivities for interated circuit design are typically given in ohm-centimetre units, although tables often provide ρ in ohm-metres or microhm-centimetres.

The value in ohm-metres is 1/100 the value in ohm-centimetres, and

$$\rho\ (\Omega\cdot\text{m}) = \left(\frac{1}{100}\right) \times (\text{value in } \Omega\cdot\text{cm})$$

Similarly:

$$\rho\ (\mu\Omega\cdot\text{cm}) = (10^6) \times (\text{value in } \Omega\cdot\text{cm})$$

For example, the resistivity of copper is

$$1.723 \times 10^{-6}\ \Omega\cdot\cancel{\text{cm}}\left[\frac{1\text{ m}}{100\ \cancel{\text{cm}}}\right] = 1.723 \times 10^{-8}\ \Omega\cdot\text{m}$$

For comparison purposes, typical values of ρ in ohm-centimetres for conductors, semiconductors, and insulators are provided in Table 3.2.

TABLE 3.2

Comparing levels of ρ in Ω cm.

Conductor	Semiconductor	Insulator
Copper 1.723×10^{-6}	Ge 50 Si 200×10^{3} GaAs 70×10^{6}	Typically 10^{15}

In particular, note the magnitude of difference between conductors and insulators (10^{21})—a huge difference. Resistivities of semiconductors cover a wide range. However, they all differ by a factor of a million or more from both conductors and insulators.

3.3 Wire Tables

The wire table was designed primarily to standardize the size of wire produced by manufacturers throughout North America. As a result, the manufacturer has a larger market and the consumer knows that standard wire sizes will always be available. The table was designed to assist the user in every way possible; in Canada, it usually includes data such as the cross-sectional area in cm^2, diameter in cm, ohms per 305 m (1000 feet) at 20°C, and weight per 305 m.

The American Wire Gauge (AWG) sizes are given in Table 3.3 for solid round copper wire.

The chosen sizes have an interesting relationship: For every drop of 3 gauge numbers, the area is almost exactly quadrupled; and for every drop 10 gauge numbers, the area increases by a factor of very close to 10.

Examining Eq. (3.1), we note also that *doubling the area cuts the resistance in half, and increasing the area by a factor of 10 decreases the resistance to 1/10 the original, everything else kept constant*.

3.4 Temperature Effects

Temperature has a significant effect on the resistance of conductors, semiconductors, and insulators.

Conductors

In conductors there is a large number of free electrons, and any introduction of thermal energy will have little impact on the total number of free electrons. In fact the thermal energy will only increase the intensity of the random motion of the ions within the material and make it increasingly difficult for a general drift of electrons in any one direction to be established. The result is that

for good conductors, an increase in temperature will result in an increase in the resistance level. As a result, conductors have a positive temperature coefficient.

TABLE 3.3

American Wire Gauge (AWG) sizes.

AWG	Diameter (cm)	Cross-sectional Area (cm^2)	Ohms/305 m @20°C
4/0	1.175	1.084	0.049
3/0	1.046	0.859	0.0618
2/0	0.931	0.681	0.078
1/0	0.829	0.5398	0.0983
2	0.658	0.3400	0.1563
4	0.522	0.2140	0.2485
6	0.414	0.1346	0.3951
8	0.328	0.0845	0.6282
10	0.260	0.0531	0.9989
12	0.206	0.0333	1.588
14	0.164	0.0211	2.525
16	0.130	0.0132	4.016
18	0.103	0.0083	6.385
20	0.0816	0.0052	10.15
22	0.0647	0.0033	16.14
24	0.0513	0.0021	25.67
26	0.0407	0.00130	40.81
28	0.0323	0.00082	64.9
30	0.0256	0.00051	103.2
32	0.0203	0.00032	164.1
34	0.0161	0.00020	260.9
36	0.0128	0.00013	414.8
38	0.0101	0.00008	659.6
40	0.00802	0.00005	1049

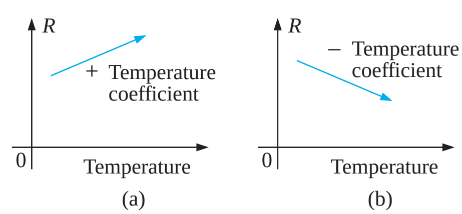


FIG. 3.3
(a) Positive temperature coefficient—conductors; (b) negative temperature coefficient—semiconductors.

The plot of Fig. 3.3(a) has a positive temperature coefficient.

Semiconductors

In semiconductors an increase in temperature will give a measure of thermal energy to the system that will result in an increase in the number of free carriers in the material for conduction. The result is that

for semiconductor materials, an increase in temperature will result in a decrease in the resistance level. As a result, semiconductors have negative temperature coefficients.

The thermistor and photoconductive cell of Sections 3.9 and 3.10 of this chapter are excellent examples of semiconductor devices with negative temperature coefficients. The plot of Fig. 3.3(b) has a negative temperature coefficient.

Insulators

As with semiconductors, an increase in temperature will result in a decrease in the resistance of an insulator. The result is a negative temperature coefficient.

Inferred Absolute Temperature

Figure 3.4 reveals that for copper (and most other metallic conductors), the resistance increases almost linearly (in a straight-line relationship) with an increase in temperature. Since temperature can have such a strong effect on the resistance of a conductor, it is important that we have some method of determining the resistance at any temperature within operating limits. An equation for this purpose can be obtained by approximating the curve of Fig. 3.4 by the straight dashed line that intersects the temperature scale at −234.5°C. Although the actual curve extends to **absolute zero** (−273.15°C, or 0 K), the straight-line approximation is quite accurate for the normal operating temperature range. At two different temperatures, T_1 and T_2, the resistance of copper is R_1 and R_2, as indicated on the curve. Using a property of similar triangles, we may develop a mathematical relationship between these values of resistances at different temperatures. Let x equal the distance from −234.5°C to T_1 and y the distance from −234.5°C to T_2, as shown in Fig. 3.4. From similar triangles,

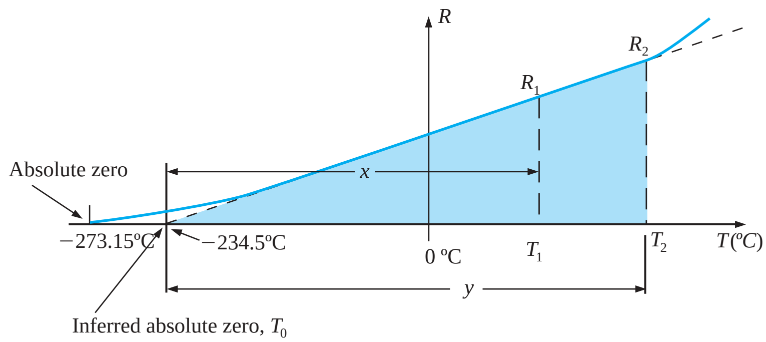


FIG. 3.4
Effect of temperature on the resistance of copper.

$$\frac{x}{R_1} = \frac{y}{R_2}$$

or

$$\frac{234.5 + T_1}{R_1} = \frac{234.5 + T_2}{R_2} \qquad (3.2)$$

TABLE 3.4
Inferred absolute temperatures.

Material	°C
Silver	−243
Copper	−234.5
Gold	−274
Aluminum	−236
Tungsten	−204
Nickel	−147
Iron	−162
Nichrome	−2 250
Constantan	−125 000

The temperature of −234.5°C is called the *inferred absolute temperature* of copper. For different conducting materials, the intersection of the straight-line approximation will occur at different temperatures. A few typical values are listed in Table 3.4.

The minus sign does not appear with the inferred absolute temperature on either side of Eq. (3.2) because x and y are the *distances* from −234.5°C to T_1 and T_2, respectively, and therefore are simply magnitudes. For T_1 and T_2 less than zero, x and y are less than $|-234.5°C|$ and the distances are the differences between the inferred absolute temperature and the temperature of interest.

Equation (3.2) can easily be adapted to any material by inserting the proper inferred absolute temperature. It may therefore be written as follows:

$$\frac{|T_0| + T_1}{R_1} = \frac{|T_0| + T_2}{R_2} \tag{3.3}$$

where T_0 indicates that the inferred absolute temperature of the material involved is inserted as a positive value in the equation. In general, therefore, associate the sign only with T_1 and T_2.

EXAMPLE 3.3

If the resistance of a copper wire is 50 Ω at 20°C, what is its resistance at 100°C (boiling point of water)?

Solution: Eq. (3.2):

$$\frac{234.5°\text{C} + 20°\text{C}}{50\ \Omega} = \frac{234.5°\text{C} + 100°\text{C}}{R_2}$$

$$R_2 = \frac{(50\ \Omega)(334.5°\text{C})}{254.5°\text{C}} = \mathbf{65.72\ \Omega}$$

EXAMPLE 3.4

If the resistance of a copper wire at freezing (0°C) is 30 Ω, what is its resistance at −40°C?

Solution: Eq. (3.2):

$$\frac{234.5°\text{C} + 0}{30\ \Omega} = \frac{234.5°\text{C} - 40°\text{C}}{R_2}$$

$$R_2 = \frac{(30\ \Omega)(194.5°\text{C})}{234.5°\text{C}} = \mathbf{24.88\ \Omega}$$

EXAMPLE 3.5

If the resistance of an aluminum wire at room temperature (20°C) is 100 mΩ (measured by a milliohmmeter), at what temperature will its resistance increase to 120 mΩ?

Solution: Eq. (3.3):

$$\frac{236°\text{C} + 20°\text{C}}{100\ \text{m}\Omega} = \frac{236°\text{C} + T_2}{120\ \text{m}\Omega}$$

and

$$T_2 = 120\ \text{m}\Omega\left(\frac{256°\text{C}}{100\ \text{m}\Omega}\right) - 236°\text{C}$$

$$T_2 = \mathbf{71.2°C}$$

Temperature Coefficient of Resistance

There is a second popular equation for calculating the resistance of a conductor at different temperatures. Defining

$$\alpha_{20} = \frac{1}{|T_0| + 20°\text{C}} \qquad (\Omega/°\text{C}/\Omega) \tag{3.4}$$

TABLE 3.5

Temperature coefficient of resistance for various conductors at 20°C.

Material	Temperature Coefficient (α_{20})
Silver	0.003 8
Copper	**0.003 93**
Gold	0.003 4
Aluminum	0.003 91
Tungsten	0.005
Nickel	0.006
Iron	0.005 5
Constantan	0.000 008
Nichrome	0.000 44

as the *temperature coefficient of resistance* at a temperature of 20°C, and R_{20} as the resistance of the sample at 20°C, the resistance R at a temperature T is determined by

$$R = R_{20}[1 + \alpha_{20}(T - 20°C)] \tag{3.5}$$

The values of α_{20} for different materials have been evaluated, and a few are listed in Table 3.5.

Equation (3.5) can be written in the following form:

$$\alpha_{20} = \frac{\dfrac{R - R_{20}}{T - 20°C}}{R_{20}} = \frac{\dfrac{\Delta R}{\Delta T}}{R_{20}}$$

from which the units of $\Omega/°C/\Omega$ for α_{20} are defined.

Since $\Delta R/\Delta T$ is the slope of the curve of Fig. 3.4, we can conclude that

the higher the temperature coefficient of resistance for a material, the more sensitive the resistance level to changes in temperature.

Looking at Table 3.5, we find that copper is more sensitive to temperature variations than is silver, gold, or aluminum, although the differences are quite small. The slope defined by α_{20} for constantan is so small the curve is almost horizontal.

Since R_{20} of Eq. (3.5) is the resistance of the conductor at 20°C and $T - 20°C$ is the change in temperature from 20°C, Eq. (3.5) can be written in the following form:

$$R = \rho\frac{l}{A}[1 + \alpha_{20}\,\Delta T] \tag{3.6}$$

providing an equation for resistance in terms of all the controlling parameters.

PPM/°C

For resistors, as for conductors, resistance changes with a change in temperature. The specification is normally provided in parts per million per degree Celsius (PPM/°C). This gives an immediate indication of the sensitivity level of the resistor to temperature. For resistors, a 5000-PPM level is considered high, whereas 20 PPM is quite low. A 1000-PPM/°C characteristic reveals that a 1° change in temperature will result in a change in resistance equal to 1000 PPM, or 1000/1 000 000 = 1/1000 of its nameplate value—not a significant change for most applications. However, a 10° change would result in a change equal to 1/100 (1%) of its nameplate value, which is becoming significant. The concern, therefore, lies not only with the PPM level but with the range of expected temperature variation.

In equation form, the change in resistance is given by

$$\Delta R = \frac{R_{\text{nominal}}}{10^6}(\text{PPM})(\Delta T) \tag{3.7}$$

where R_{nominal} is the nameplate value of the resistor at room temperature and ΔT is the change in temperature from the reference level of 20°C.

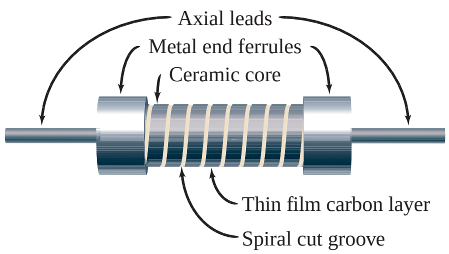


FIG. 3.5

EXAMPLE 3.6

For a 1-kΩ carbon composition resistor with a PPM of 2500, determine the resistance at 60°C.

Solution:

$$\Delta R = \frac{1000\ \Omega}{10^6}(2500)(60°\text{C} - 20°\text{C})$$
$$= 100\ \Omega$$

and
$$R = R_{\text{nominal}} + \Delta R = 1000\ \Omega + 100\ \Omega$$
$$= \mathbf{1100\ \Omega}$$

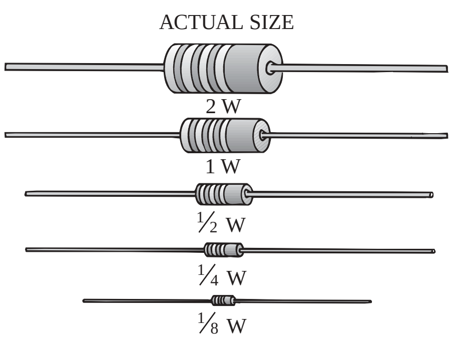


FIG. 3.6
Fixed composition resistors of different wattage ratings.

3.5 TYPES OF RESISTORS

Fixed Resistors

Resistors are made in many forms, but all belong in either of two groups: fixed or variable. The most common of the low-wattage, fixed-type resistors is the thin film carbon resistor. The basic construction is shown in Fig. 3.5. Resistors of this type are readily available in values ranging from 2.7 Ω to 22 MΩ.

Resistors rated for higher powers need to be larger to withstand greater heat dissipation. The relative sizes of the molded composition resistors for different power ratings (wattage) are shown in Fig. 3.6.

The temperature-versus-resistance curves for a 10-kΩ and a 0.5-MΩ resistor are shown in Fig. 3.7. Note the small percent resistance change in the normal temperature operating range. Several other types of fixed resistors using high resistance wire or metal films are shown in Fig. 3.8.

The miniaturization of parts—used quite extensively in computers—requires that resistances of different values be placed in very small packages. Some examples appear in Fig. 3.9.

For use with printed circuit boards, fixed resistor networks in a variety of configurations are available in miniature packages, such as those shown in Fig. 3.10. The figure includes a photograph of three different casings and the internal resistor configuration for the single in-line structure to the right.

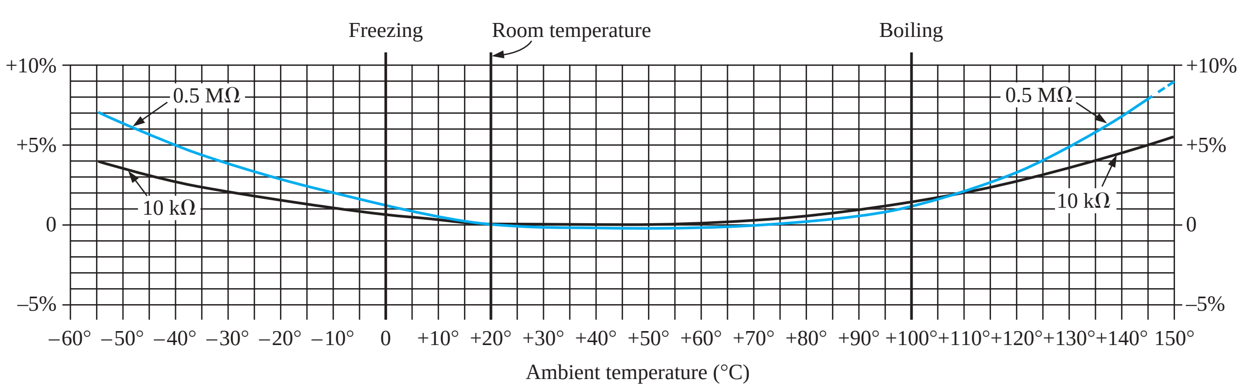
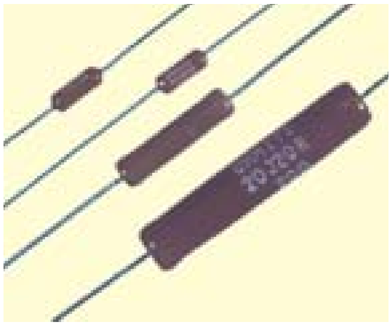
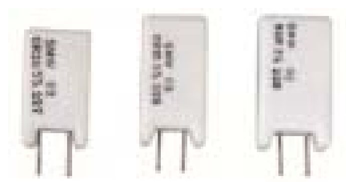


FIG. 3.7
Curves showing percentage temporary resistance changes from +20°C values. (Courtesy of Allen-Bradley Co.)



(a) Vitreous conformal wire resistor



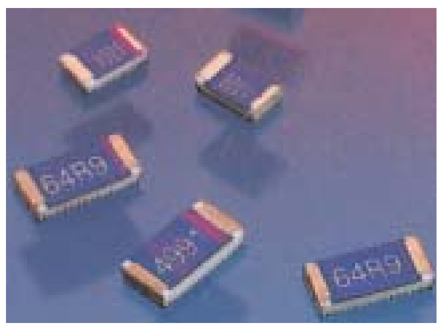
(b) Power wire-wound stand-up cemented leaded fixed resistors

(c) High precision and ultra high precision metal film leaded fixed resistors

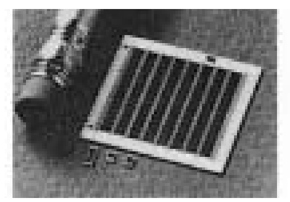
FIG. 3.8

Resistors. [Part (a) courtesy of Ohmite Manufacturing Co. Part (b) and (c) courtesy of Philips Components Inc.]

(a) Surface mount power resistors ideal for printed circuit boards



(b) Surface mount resistors



(c) Thick-film chip resistors for design flexiblity with hybrid circuitry. Pretinned, gold and silver electrodes available. Operating temperature range −55° to +150°C

FIG. 3.9

Miniature fixed resistors. [Part (a) courtesy of Ohmite Manufacturing Co. Parts (b) courtesy of Philips Components Inc. (c) courtesy of Vishay Dale Electronics, Inc.]

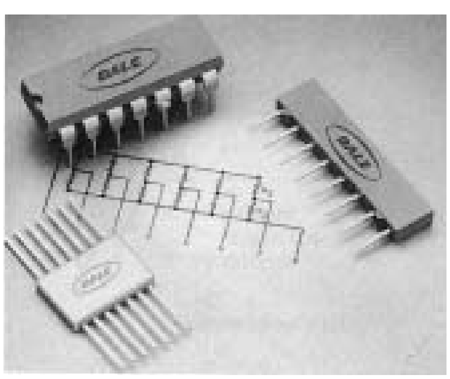


FIG. 3.10

Thick-film resistor networks. (Courtesy of Vishay Dale Electronics, Inc.)

Variable Resistors

Variable resistors, as the name suggests, have a terminal resistance that can be varied by turning a dial, knob, or screw. They can have two or three terminals, but most have three terminals. If the two- or three-terminal device is used as a variable resistor, it is usually referred to as a **rheostat**. If the three-terminal device is used for controlling potential levels, it is then commonly called a **potentiometer**. Even though a three-terminal device can be used as a rheostat or potentiometer (depending on how it is connected), it is typically called a *potentiometer* in trade magazines or when requested for a particular application.

The symbol for a three-terminal potentiometer appears in Fig. 3.11(a). When used as a variable resistor (or rheostat), it can be hooked up in one of two ways, as shown in Fig. 3.11(b) and (c). In Fig. 3.11(b), points *a* and *b* are hooked up to the circuit, and the remaining terminal is left hanging. The resistance introduced is determined by that portion of the resistive element between points *a* and *b*. In Fig. 3.11(c), the resistance is again between points *a* and *b*, but now the remaining resistance is

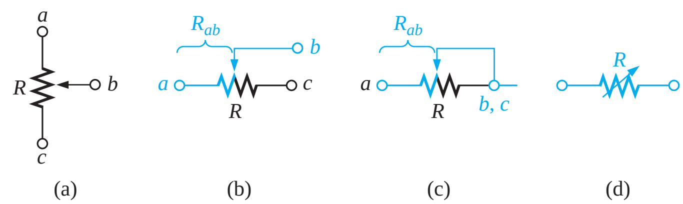
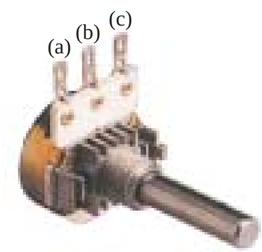
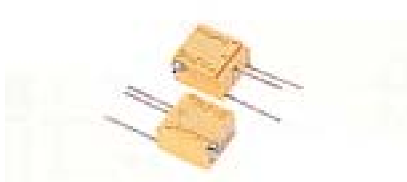


FIG. 3.11
Potentiometer: (a) symbol; (b) and (c) rheostat connections; (d) rheostat symbol.



(a) Cermet control potentiometer

(b) Cermet single turn trimming potentiometer



(c) Cermet multiturn trimming potentiometer

FIG. 3.12
Potentiometer. (Courtesy of Phillips Components Inc.)

"shorted-out" (effectively removed) by the connection from *b* to *c*. The universally accepted symbol for a rheostat appears in Fig. 3.11(d).

Most potentiometers have three terminals in the relative positions shown in Fig. 3.12. The knob, dial, or screw in the centre of the housing controls the motion of a wiper arm, a contact that can move along the resistive element connected between the outer two terminals. The contact is connected to the centre terminal, establishing a resistance from the movable contact to each outer terminal.

The resistance between the outside terminals a and c of Fig. 3.13(a) (and Fig. 3.12) is always fixed at the full rated value of the potentiometer, regardless of the position of the wiper arm b.

In other words, the resistance between terminals *a* and *c* of Fig. 3.13(a) for a 1-MΩ potentiometer will always be 1 MΩ, no matter how we turn the control element and move the contact. In Fig. 3.13(a) the centre contact is not part of the network configuration.

The resistance between the wiper arm and either outside terminal can be varied from a minimum of 0 Ω to a maximum value equal to the full rated resistance value of the potentiometer.

In Fig. 3.13(b) the wiper arm has been placed 1/4 of the way down from point *a* to point *c*. The resulting resistance between points *a* and *b* will therefore be 1/4 of the total, or 250 kΩ (for a 1-MΩ potentiometer), and the resistance between *b* and *c* will be 3/4 of the total, or 750 kΩ.

The sum of the resistances between the wiper arm and each outside terminal will equal the full rated resistance of the potentiometer.

This was demonstrated by Fig. 3.13(b), where 250 kΩ + 750 kΩ = 1 MΩ. Specifically:

$$R_{ac} = R_{ab} + R_{bc} \tag{3.8}$$

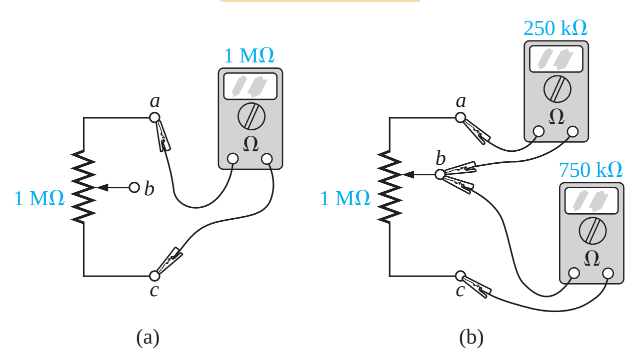


FIG. 3.13
Terminal resistance of a potentiometer: (a) between outside terminals; (b) among all three terminals.

Therefore, as the resistance from the wiper arm to one outside contact increases, the resistance between the wiper arm and the other outside terminal must decrease accordingly. For example, if R_{ab} of a 1-kΩ potentiometer is 200 Ω, then the resistance R_{bc} must be 800 Ω. If R_{ab} is further decreased to 50 Ω, then R_{bc} must increase to 950 Ω, and so on.

The molded carbon composition potentiometer is typically applied in networks with smaller power demands, and it ranges in size from 20 Ω to 22 MΩ (maximum values). Other commercially available potentiometers appear in Fig. 3.14.

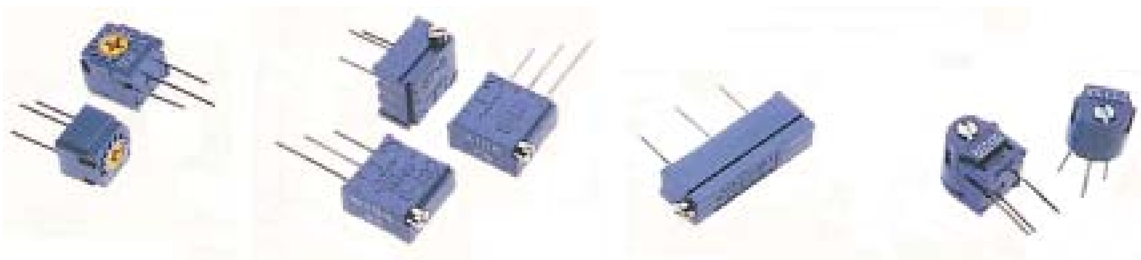


FIG. 3.14
Trimming potentiometers. (Courtesy of Phillips Components Inc.)

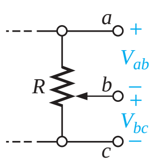


FIG. 3.15
Potentiometer control of voltage levels.

When the device is used as a potentiometer, the connections are as shown in Fig. 3.15. It can be used to control the level of V_{ab}, V_{bc}, or both, depending on the application. Additional discussion of the potentiometer in a loaded situation can be found in the chapters that follow.

3.6 Colour Coding and Standard Resistor Values

Many resistors, fixed or variable, are large enough to have their resistance in ohms printed on the casing. Some, however, are too small to have numbers printed on them, so a system of colour coding is used. For the fixed carbon film resistor, four or five colour bands are printed on one end of the outer casing, as shown in Fig. 3.16. Each colour has the numerical value indicated in Table 3.6. The colour bands are always read from the end that has the band closest to it, as shown in Fig. 3.16. The first and second bands represent the first and second digits, respectively. The third band determines the power-of-10 multiplier for the first two digits (actually, the number of zeros that follow the second digit) or a multiplying factor if gold or silver. The fourth band is the manufacturer's tolerance, which is an indication of the precision with which the resistor was made. If the fourth band is omitted, the tolerance is assumed to be $\pm 20\%$. Sometimes a fifth band shows a reliability factor, which gives the percentage of failure per 1000 hours of use. For instance, a 1% failure rate would reveal that one out of every 100 (or 10 out of every 1000) will fail to fall within the tolerance range after 1000 hours of use.

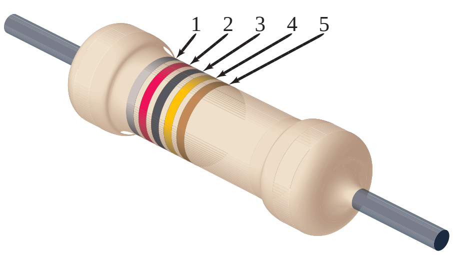


FIG. 3.16
Colour coding—fixed molded composition resistor.

One might expect that resistors would be available for a full range of values such as 10 Ω, 20 Ω, 30 Ω, 40 Ω, 50 Ω, and so on. However, this is not the case—some typical commercial values are 27 Ω, 56 Ω, and 68 Ω. This may seem odd, but there is a reason for the chosen values. It can be demonstrated by examining the list of standard values of commercially available resistors in Table 3.7. The values in boldface blue are available with 5%, 10%, and 20% tolerances, making them the most common of the commercial variety. The values in boldface black

TABLE 3.6
Resistor colour coding.

Bands 1 and 2 Value	Band 3 Multiplier	Band 4 Tolerance	Band 5 Reliability
0 Black	0 Black	5% Gold	1% Brown
1 Brown	1 Brown	10% Silver	0.1% Red
2 Red	2 Red	20% No band	0.01% Orange
3 Orange	3 Orange		0.001% Yellow
4 Yellow	4 Yellow		
5 Green	5 Green		
6 Blue	6 Blue		
7 Violet	7 Violet		
8 Grey	8 Grey		
9 White	9 White		
	0.1 Gold		
	0.01 Silver		

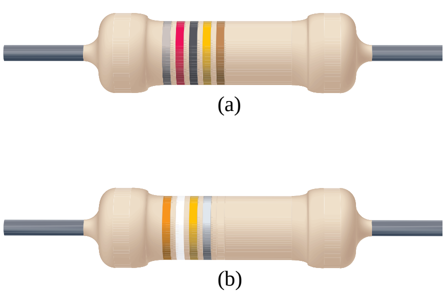


FIG. 3.17
Resistors for Example 3.7.

EXAMPLE 3.7

Find the range in which a resistor having the following colour bands (Fig. 3.17) must exist to satisfy the manufacturer's tolerance:

a.

1st band	2nd band	3rd band	4th band	5th band
Grey	Red	Black	Gold	Brown
8	2	0	±5%	1%

82 Ω ± 5% (1% reliability)

Since 5% of 82 = 4.10, the resistor should be within the range 82 Ω ± 4.10 Ω, or *between 77.90 and 86.10 Ω*.

b.

1st band	2nd band	3rd band	4th band	5th band
Orange	White	Gold	Silver	No colour
3	9	0.1	±10%	

3.9 Ω ± 10% = 3.9 ± 0.39 Ω

The resistor should lie somewhere *between 3.51 and 4.29 Ω*.

are typically available with 5% and 10% tolerances, and those in normal print are available only in the 5% variety. If we separate the values available into tolerance levels, we have Table 3.8, which clearly reveals how few are available up to 100 Ω with 20% tolerances.

An examination of the impact of the tolerance level will now help explain the choice of numbers for the commercial values. Take the sequence 47 Ω–68 Ω–100 Ω, which are all available with 20% tolerances. In Fig. 3.18(a), the tolerance band for each has been determined and plotted on a single axis. Take note that, with this tolerance (which is all the manufacturer will guarantee), the full range of resistor values is available from 37.6 Ω to 120 Ω. In other words, the manufacturer is guaranteeing the full range, using the tolerances to fill in the gaps. Dropping to the 10% level introduces the 56-Ω and 82-Ω resistors to fill in the gaps, as shown in Fig. 3.18(b). Dropping to the 5% level would require additional resistor values to fill in the gaps. In total, therefore, the resistor values were chosen to ensure that the full range was covered, as determined by the tolerances used. Of course, if a specific value is desired but is not one of the standard values, combinations of standard

TABLE 3.7
Standard values of commercially available resistors.

Ohms (Ω)					Kilohms (kΩ)		Megohms (MΩ)	
0.10	**1.0**	**10**	**100**	**1000**	**10**	**100**	**1.0**	**10.0**
0.11	1.1	11	110	1100	11	110	1.1	11.0
0.12	**1.2**	**12**	**120**	**1200**	**12**	**120**	**1.2**	**12.0**
0.13	1.3	13	130	1300	13	130	1.3	13.0
0.15	**1.5**	**15**	**150**	**1500**	**15**	**150**	**1.5**	**15.0**
0.16	1.6	16	160	1600	16	160	1.6	16.0
0.18	**1.8**	**18**	**180**	**1800**	**18**	**180**	**1.8**	**18.0**
0.20	2.0	20	200	2000	20	200	2.0	20.0
0.22	**2.2**	**22**	**220**	**2200**	**22**	**220**	**2.2**	**22.0**
0.24	2.4	24	240	2400	24	240	2.4	
0.27	**2.7**	**27**	**270**	**2700**	**27**	**270**	**2.7**	
0.30	3.0	30	300	3000	30	300	3.0	
0.33	**3.3**	**33**	**330**	**3300**	**33**	**330**	**3.3**	
0.36	3.6	36	360	3600	36	360	3.6	
0.39	**3.9**	**39**	**390**	**3900**	**39**	**390**	**3.9**	
0.43	4.3	43	430	4300	43	430	4.3	
0.47	**4.7**	**47**	**470**	**4700**	**47**	**470**	**4.7**	
0.51	5.1	51	510	5100	51	510	5.1	
0.56	**5.6**	**56**	**560**	**5600**	**56**	**560**	**5.6**	
0.62	6.2	62	620	6200	62	620	6.2	
0.68	**6.8**	**68**	**680**	**6800**	**68**	**680**	**6.8**	
0.75	7.5	75	750	7500	75	750	7.5	
0.82	**8.2**	**82**	**820**	**8200**	**82**	**820**	**8.2**	
0.91	9.1	91	910	9100	91	910	9.1	

TABLE 3.8
Standard values and their tolerances.

±5%	±10%	±20%
10	**10**	**10**
11		
12	**12**	
13		
15	**15**	**15**
16		
18	**18**	
20		
22	**22**	**22**
24		
27	**27**	
30		
33	**33**	**33**
36		
39	**39**	
43		
47	**47**	**47**
51		
56	**56**	
62		
68	**68**	**68**
75		
82	**82**	
91		

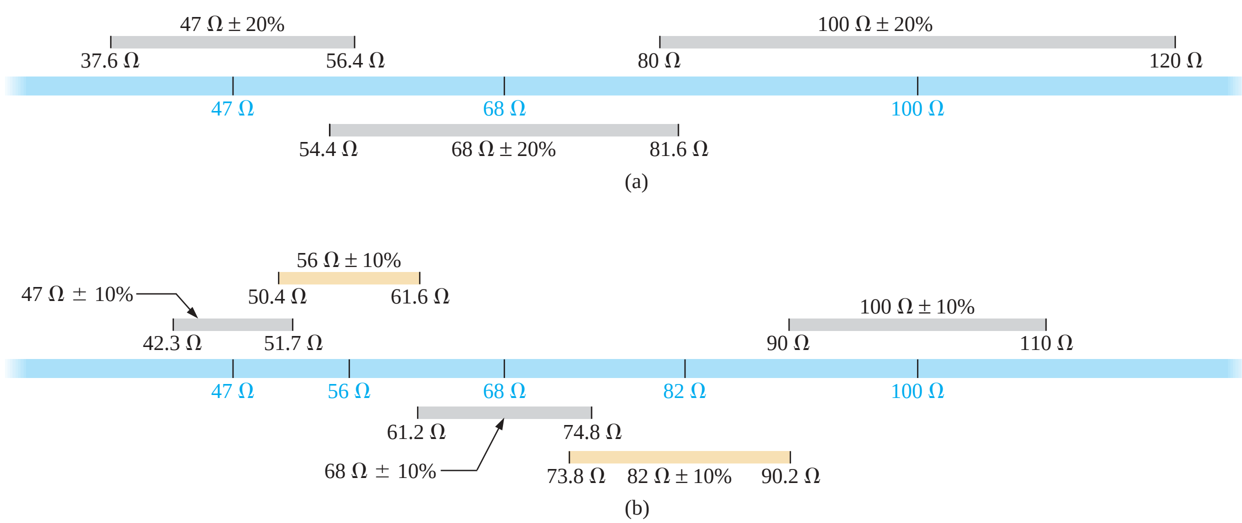


FIG. 3.18
Guaranteeing the full range of resistor values for the given tolerance: (a) 20%; (b) 10%.

values will often result in a total resistance very close to the desired level. If this approach is still not satisfactory, a potentiometer can be set to the exact value and then inserted in the network.

Throughout the text you will find that many of the resistor values are not standard values. This was done to reduce the mathematical complexity, which might cloud the procedure being introduced. In the problem sections, however, standard values are frequently used to help you to become familiar with the commercial values available.

L U M I N A R I E S

German (Lenthe, Berlin)
Electrical Engineer
Telegraph Manufacturer:
Siemens & Halske AG

FIG. 3.19
Werner von Siemens (1816–1892)

Siemens developed an *electroplating process* during a brief stay in prison for acting as a second in a duel between fellow officers of the Prussian army. Inspired by the electric telegraph invented by Sir Charles Wheatstone in 1837, he improved on the design and proceeded to lay cable with the help of his brother Carl across the Mediterranean and from Europe to India. His inventions included the first *self-excited generator*, which depended on the *residual* magnetism of its electromagnet rather than an inefficient permanent magnet. In 1888 he was raised to the rank of nobility with the addition of *von* to his name. The current firm of Siemens AG has manufacturing outlets in 35 countries with sales offices in 125 countries.

Bettman Archives Photo Number 336.19

3.7 Conductance

Sometimes, instead of resistance it is useful to know how well a material will *conduct* current. This property is known as **conductance**, and is defined as the reciprocal of resistance. Conductance has the symbol *G*, and is measured in **siemens (S)** (note Fig. 3.19). In equation form, conductance is

$$G = \frac{1}{R} \qquad \text{(siemens, S)} \tag{3.9}$$

A resistance of 1 MΩ is equivalent to a conductance of 10^{-6} S, and a resistance of 10 Ω is equivalent to a conductance of 10^{-1} S. The larger the conductance, therefore, the less the resistance and the greater the conductivity.

In equation form, the conductance is determined by

$$G = \frac{A}{\rho l} \qquad \text{(S)} \tag{3.10}$$

indicating that increasing the area or decreasing either the length or the resistivity will increase the conductance.

EXAMPLE 3.8

What is the relative increase or decrease in conductivity of a conductor if the area is reduced by 30% and the length is increased by 40%? The resistivity is fixed.

Solution: Eq. (3.10):

$$G_i = \frac{A_i}{\rho_i l_i}$$

with the subscript *i* for the initial value. Using the subscript *n* for new value:

$$G_n = \frac{A_n}{\rho_n l_n} = \frac{0.70A_i}{\rho_i(1.4l_i)} = \frac{0.70}{1.4}\frac{A_i}{\rho_i l_i} = \frac{0.70\,G_i}{1.4}$$

and $\quad \mathbf{G_n = 0.5G_i}$

3.8 Ohmmeters

The **ohmmeter** is an instrument used to measure resistance and to perform other useful tasks:

1. ***Measure the resistance of individual or combined elements***
2. ***Detect open-circuit (high-resistance) and short-circuit (low-resistance) situations***
3. ***Check continuity of network connections and identify wires of a multilead cable***

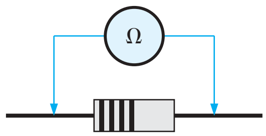


FIG. 3.20
Measuring the resistance of a single element.

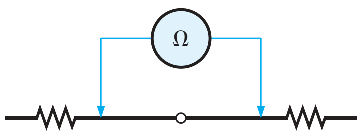


FIG. 3.21
Checking the continuity of a connection.

For most applications, the ohmmeter used is the ohmmeter section of a VOM or DMM. In general, the resistance of a resistor can be measured by simply connecting the two leads of the meter across the resistor, as shown in Fig. 3.20. There is no need to be concerned about which lead goes on which end; the result will be the same in either case since resistors offer the same resistance to the flow of current in either direction. When measuring the resistance of a single resistor, it is usually best to remove the resistor from the network before making the measurement. If this is difficult or impossible, at least one end of the resistor must not be connected to the network, or the reading may include the effects of the other elements of the system.

If the two leads of the meter are touching in the ohmmeter mode, the resulting resistance is zero. A connection can be checked as shown in Fig. 3.21 by simply hooking up the meter to both sides of the connection. If the resistance is zero, the connection is secure. If it is other than zero, it could be a poor connection, and, if it is infinite, there is no connection at all.

If one wire of a harness is known, a second can be found as shown in Fig. 3.22. Simply connect the end of the known lead to the end of any other lead. When the ohmmeter indicates zero ohms (or very low resistance), the second lead has been identified. The above procedure can also be used to determine the first known lead by simply connecting the meter to any wire at one end and then touching all the leads at the other end until a zero-ohm indication is obtained.

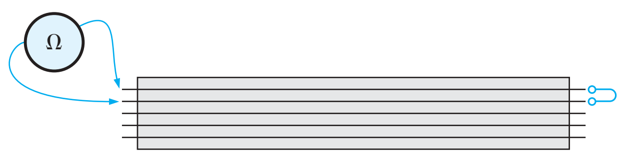


FIG. 3.22
Identifying the leads of a multilead cable.

3.9 THERMISTORS

The **thermistor** is a two-terminal semiconductor device whose resistance, as the name suggests, is temperature sensitive. A representative characteristic appears in Fig. 3.23 with the graphic symbol for the device. Note the nonlinearity of the curve and the drop in specific resistance from about 5000 Ω·cm to 100 Ω·cm for an increase in temperature from 20°C to 100°C. The decrease in resistance with an increase in temperature indicates a negative temperature coefficient.

The temperature of the device can be changed internally or externally. An increase in current through the device will raise its temperature, causing a drop in its terminal resistance. Any externally applied heat source will result in an increase in its body temperature and a drop in resistance. This type of action (internal or external) lends itself well to control mechanisms. Many different types of thermistors are shown in Fig. 3.24. Materials used to make thermistors include oxides of cobalt, nickel, strontium, and manganese.

Note the use of a log scale (to be discussed in Chapter 21) in Fig. 3.23 for the vertical axis. The log scale makes it possible to show a wider range of specific resistance levels than a linear scale such as the horizontal axis. Note that it extends from 0.0001 Ω·cm to 100 000 000 Ω·cm over a very short interval. The log scale is used for both the vertical and the horizontal axis of Fig. 3.25, which appears in the next section.

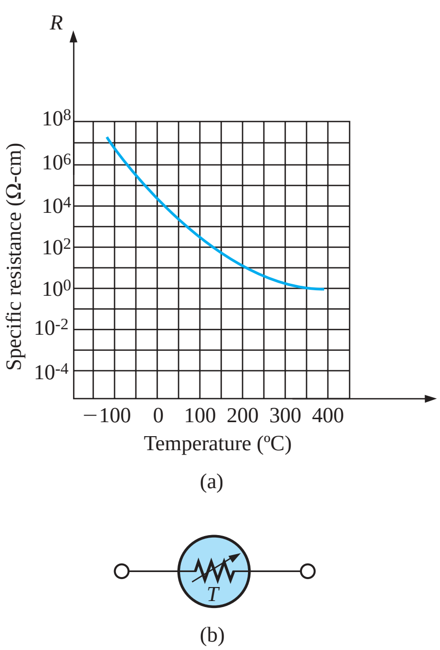


FIG. 3.23
Thermistor: (a) characteristics; (b) symbol.

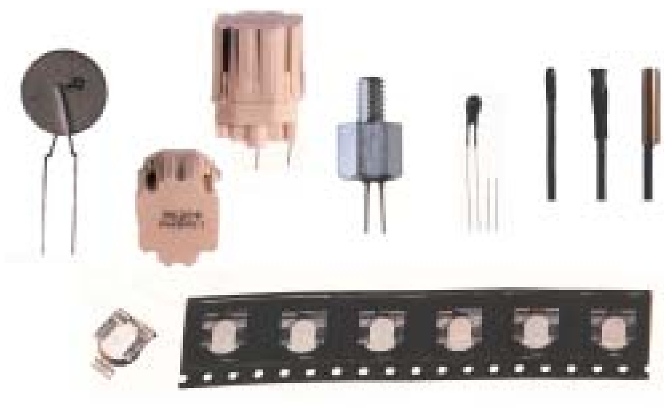


FIG. 3.24
NTC (negative temperature coefficient) and PTC (positive temperature coefficient) thermistors. (Courtesy of Philips Components Inc.)

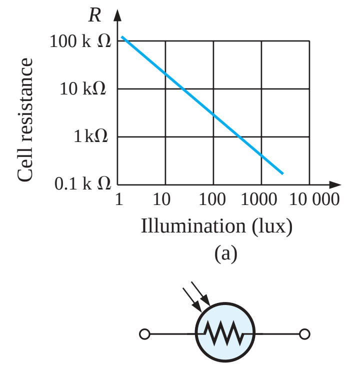


FIG. 3.25
Photoconductive cell: (a) characteristics; (b) symbol.

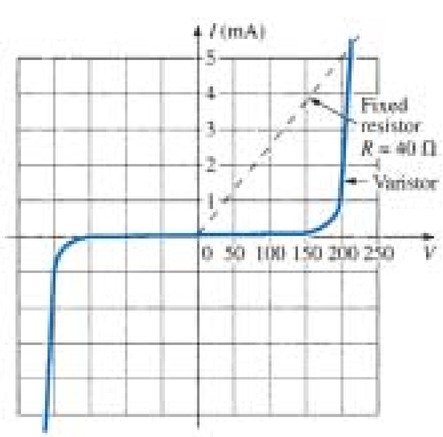
FIG. 3.26
Street lighting photocontrol that uses a photoconductive cell (visible in the window of the casings). (Courtesy of Precision.)

3.10 PHOTOCONDUCTIVE CELLS

The **photoconductive cell** is a two-terminal semiconductor device with a terminal resistance that is determined by the intensity of the incident light on its exposed surface. As the applied illumination increases in intensity, the energy state of the surface electrons and atoms increases. The result is that the number of "free carriers" increases and the resistance drops. A typical set of characteristics and the photoconductive cell's graphic symbol appear in Fig. 3.25. Note the negative illumination coefficient. A commonly used photoconductive cell appears in Fig. 3.26.

3.11 VARISTORS

Varistors are voltage-dependent, nonlinear resistors used to suppress high-voltage transients. In other words, their characteristics limit the voltage that can appear across the terminals of a sensitive device or system. At a particular "firing voltage," the current rises rapidly but the voltage is limited to a level just above this firing potential, as shown in Fig. 3.27 (a). In other words, the magnitude of the voltage that can appear across this device cannot be greater than the level defined by its characteristics. Through proper design techniques this device can therefore limit the voltage appearing across sensitive regions of a network. The current is simply limited by the network to which it is connected. A photograph of a number of commercial units appears in Fig. 3.27 (b).



(a)

(b)

FIG. 3.27
(a) A typical characteristic curve for a varistor; (b) zinc-oxide varistors for overvoltage protection (Courtesy of Philips Components, Inc.)

PROBLEMS

SECTION 3.2 Resistance: Metric Units

*1. **a.** What is the resistance of a copper bus-bar with the dimensions shown in Fig. 3.28 ($T = 20°C$)?
b. Repeat (a) for aluminum and compare the results.
c. Without working out the numerical solution, determine whether the resistance of the bar (aluminum or copper) will increase or decrease with an increase in length.
d. Repeat (c) for an increase in cross-sectional area.

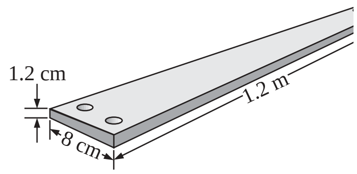


FIG. 3.28
Problem 1.

2. Using metric units, determine the length of a #12 AWG copper wire that has a resistance of 0.2 Ω and a diameter of 0.13 cm.

3. Determine the resistance of 100.0 m of #28 AWG copper telephone wire at 20°C.

4. Report problem 3 for temperatures of
a. −40°C. **b.** +40°C.

5. A 2.2-Ω resistor is to be made of Nichrome wire. If the available wire is 0.8 mm in diameter, how much wire is required?

6. A 0.25-Ω resistor is to be made from tantalum wire of #18 gauge. What length of wire is required?

SECTION 3.3 Wire Tables

7. **a.** Using Table 3.3, find the resistance of 450 m of #12 and #14 AWG wires.
b. Compare the resistances of the two wires.
c. Compare the cross-sectional areas of the two wires.

8. **a.** Using Table 3.3, find the resistance of 1800 m of #8 and #18 AWG wires.
b. Compare the resistances of the two wires.
c. Compare the cross-sectional areas of the two wires.

SECTION 3.4 Temperature Effects

9. The resistance of a copper wire is 2 Ω at 10°C. What is its resistance at 60°C?

10. The resistance of an aluminum bus-bar is 0.02 Ω at 0°C. What is its resistance at 100°C?

11. The resistance of a copper wire is 4 Ω at 21°C. What is its resistance at −20°C?

12. The resistance of a copper wire is 0.76 Ω at 30°C. What is its resistance at −40°C?

13. Calculate the resistance of 10 m of #18 AWG Nichrome wire at 60°C.

14. If the resistance of a silver wire is 0.04 Ω at −30°C, what is its resistance at 0°C?

*15. **a.** The resistance of a nichrome wire is 0.025 Ω at room temperature (20°C). What is its resistance at 0°C (freezing) and 100°C (boiling)?
b. For (a), determine the change in resistance for each 20° change in temperature between room temperature and 100°C and graph the results.

16. **a.** The resistance of an aluminum wire is 0.92 Ω at 4°C. At what temperature (°C) will it be 1.06 Ω?
b. At what temperature will it be 0.15 Ω?

*17. **a.** If the resistance of a 300-m length of copper wire is 10 Ω at room temperature (20°C), what will its resistance be at 50 K using Eq. (3.3)?
b. Repeat part (a) for a temperature of 38.65 K. Comment on the results obtained by reviewing the curve of Fig. 3.4.

18. A 120 V lamp has a resistance of 12Ω at 22°C. During normal operation, the resistance is 144 Ω. Find the filament temperature of the operating lamp.

19. **a.** Verify the value of α_{20} for copper in Table 3.5 by substituting the inferred absolute temperature into Eq. (3.4).
b. Using Eq. (3.5), find the temperature at which the resistance of a copper conductor will increase to 1 Ω from a level of 0.8 Ω at 20°C.

20. Using Eq. (3.5), find the resistance of an aluminum wire at 10°C if its resistance at 20°C is 0.4 Ω.

*21. Determine the resistance of a 100-m coil of #18 AWG copper wire sitting in the desert at a temperature of 45°C.

22. A 22-Ω wire-wound resistor is rated at + 150 PPM for a temperature range of −10°C to +75°C. Determine its resistance at 65°C.

23. Determine the PPM rating of the 10-kΩ resistor of Fig. 3.7 using the resistance level determined at 90°C.

24. A 1-m length of #22 AWG wire has an R of 0.24 Ω at 20°C. At 100°C R = 0.3552 Ω. Determine the material used in the wire.

SECTION 3.5 Types of Resistors

25. **a.** What is the approximate increase in size from a 1-W to a 2-W resistor?
b. What is the approximate increase in size from a 1/2-W to a 2-W resistor?

26. If the 10-kΩ resistor of Fig. 3.7 is exactly 10 kΩ at room temperature, what is its approximate resistance at −30°C and 100°C (boiling)?

27. Repeat Problem 26 at temperatures of 0°C and 75°C.

28. If the resistance between the outside terminals of a linear potentiometer is 10 kΩ, what is its resistance between the wiper (movable) arm and an outside terminal if the resistance between the wiper arm and the other outside terminal is 3.5 kΩ?

29. If the wiper arm of a linear potentiometer is one-third the way around the contact surface, what is the resistance between the wiper arm and each terminal if the total resistance is 25 kΩ?

***30.** Show the connections required to establish 4 kΩ between the wiper arm and one outside terminal of a 10-kΩ potentiometer while having only zero ohms between the other outside terminal and the wiper arm.

31. A portion of a 300-m roll of #14 AWG wire has been used. What is the length of the remaining wire if the measured resistance is 1.68 Ω?

SECTION 3.6 Colour Coding and Standard Resistor Values

32. Find the range in which a resistor having the following colour bands must exist to satisfy the manufacturer's tolerance:

	1st band	2nd band	3rd band	4th band
a.	green	blue	orange	gold
b.	red	red	brown	silver
c.	brown	black	black	—
d.	blue	grey	yellow	gold

33. Find the colour code for the following 10% resistors:
a. 0.1 Ω **b.** 22 Ω
c. 3300 Ω **d.** 68 kΩ
e. 1 MΩ

34. Is there an overlap in coverage between 10% resistors? That is, determine the tolerance range for a 10-Ω 10% resistor and a 12-Ω 10% resistor, and note whether their tolerance ranges overlap.

35. Repeat Problem 34 for 5% resistors of the same value.

SECTION 3.7 Conductance

36. Find the conductance of each of the following resistances:
a. 0.086 Ω **b.** 4 kΩ
c. 2.2 MΩ
Compare the three results.

37. Find the conductance of 300 m of #14 AWG wire made of
a. copper
b. aluminum
c. iron

***38.** The conductance of a wire is 100 S. If the area of the wire is increased by 2/3 and the length is reduced by the same amount, find the new conductance of the wire if the temperature remains fixed.

SECTION 3.8 Ohmmeters

39. How would you check the status of a fuse with an ohmmeter?

40. How would you determine the on and off states of a switch using an ohmmeter?

41. How would you use an ohmmeter to check the status of a light bulb?

42. Why should an ohmmeter never be used in an energized circuit?

SECTION 3.9 Thermistors

***43. a.** Find the specific resistance of the thermistor having the characteristics of Fig. 3.23 at −50°C, 50°C, and 100°C. Note that it is a log scale. If necessary, consult a reference with an expanded log scale.
b. Does the thermistor have a positive or negative temperature coefficient?
c. Is the coefficient a fixed value for the range −100°C to 400°C? Why?
d. What is the approximate rate of change of ρ with temperature at 100°C?

SECTION 3.10 Photoconductive Cells

***44. a.** Using the characteristics of Fig. 3.25, determine the resistance of the photoconductive cell at 100 and 1000 lux illumination. As in Problem 43, note that it is a log scale.
b. Does the cell have a positive or negative illumination coefficient?
c. Is the coefficient a fixed value for the range 1 to 10 000 lux? Why?
d. What is the approximate rate of change of ρ with illumination at 100 lux?

SECTION 3.11 Varistors

45. a. Referring to Fig. 3.27(a), find the terminal voltage of the device at 0.5, 1, 3, and 5 mA.
b. What is the total change in voltage for the indicated range of current levels?
c. Compare the ratio of maximum to minimum current levels above to the corresponding ratio of voltage levels.