Numeracy 1

Notes for the Teacher

Chapter 1 Planning Chart

Section	Lesson Focus	Resources
Necessary Skills	Review prerequisite skills and concepts for the chapter	Masters 1.1, 1.2; scientific calculator
1.1 Rational Numbers	Learn what a rational number is, represent, compare, and order rational numbers.	Masters 1.3, 1.7; scientific calculator; ruler; grid paper (optional)
1.2 Adding and Subtracting Rational Numbers	Estimate, calculate, and verify the sum and difference of rational numbers.	Masters 1.3, 1.8; scientific calculator
1.3 Multiplying and Dividing Rational Numbers	Estimate, calculate, and verify the product and quotient of rational numbers.	Masters 1.4, 1.9; scientific calculator
1.4 Order of Operations with Rational Numbers	Apply the correct order of operations when evaluating expressions with rational numbers.	Master 1.4; scientific calculator
1.5 Adding and Subtracting Matrices	Investigate the properties of matrices, and use them to solve problems in which data is added or subtracted.	Masters 1.5, 1.10; scientific calculator; graphing calculator (optional)
1.6 Multiplying a Matrix by a Number	Solve problems in which data in a matrix is multiplied by a scalar.	Masters 1.5, 1.11; scientific calculator
1.7 Mathematical Modelling: How Many Cereal Boxes?	Model the problem of determining how many boxes of cereal must be purchased to obtain a complete set of promotional items inserted in the boxes.	Master 1.6; scientific calculator; coins; grid paper; playing cards

Scientific calculators should be available for student use throughout the chapter.

Chapter 1 Overview

Curriculum Quote

"Mathematics learning is an active and constructive process. Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates. Learning is most likely when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and nurtures positive attitudes and sustained effort."

To be successful in mathematics, students should have facility with integers, fractions, decimals, and percents. In this chapter, these basic concepts are used to develop operations with rational numbers and matrices.

In this chapter:

- Students develop the rational number as a fraction with numerator and denominator as integers.
- Students represent rational numbers on a number line.
- Students write rational numbers in both decimal and fractional form.
- Students use the rules for integers and fractions to develop the rules for performing basic operations with rational numbers.
- Students use rational numbers in a variety of real-life problems.
- Students identify the basic properties of a matrix.
- Students use matrices to represent real-life situations.
- By creating a mathematical model, students develop a solution to a problem involving the number of items needed to complete a set of items.

By the end of this chapter, students should be able to:

- Order rational numbers on a number line.
- Round rational numbers to the nearest integer.
- Convert rational numbers between fractional and decimal forms.
- Add, subtract, multiply, divide, and use the order of operations with rational numbers.
- Represent numbers given in a rectangular array in matrix form.
- Add and subtract matrices with the same dimensions.
- Multiply the entries in a matrix by a scalar.
- Use a coin, a die, and playing cards to simulate the problem of how many cereal boxes would be needed to obtain a complete set of promotional items inserted in the boxes.
- Discuss the advantages and disadvantages of a mathematical model to solve a problem.
- Represent the results of a simulation in a table and on a graph.

Chapter 1 Necessary Skills, page 4

Suggested Materials

Master 1.1 and 1.2; scientific calculator

Building Mathematical Literacy

Mathematical words and phrases for student success in Chapter 1:

experimental model	matrix column	order of operations
integer	matrix dimensions	rational number
mathematical model	matrix row	rectangular array
matrix	number line	scalar
		simulation

Have students record unfamiliar terms in their notebooks, then use the glossary in the student book, or another reference, to research meanings. Encourage students to write definitions in their own words.

Assessment Plan

For success in Chapter 1, students need facility with the content areas addressed in the student book.

As students complete the exercises, encourage them to note areas in which they have difficulty. The chart below will help you and your students determine where they need additional practice.

Necessary Skills	Related Sections:	Additional Practice
Adding and Subtracting Integers	1.2, 1.4, 1.5	Master 1.1
Multiplying and Dividing Integers	1.3, 1.4, 1.6	Master 1.1
Adding and Subtracting Fractions	1.2, 1.4	Master 1.1
Multiplying and Dividing Fractions	1.1, 1.3, 1.4	Master 1.2
Fractions to Decimals and Decimals to Fractions	1.1	Master 1.2
Percent	1.6	Master 1.2

Students who complete all the exercises successfully are ready to begin Section 1.1. For students having difficulty, assign additional practice as needed using the appropriate Masters listed in the table.

1.1 Rational Numbers, page 12

Curriculum Correlation

Students will represent numbers in multiple ways and apply appropriate representations to solve problems.

Students will read, write, and order rational numbers.

Suggested Materials

Masters 1.3, 1.7; scientific calculator; ruler; grid paper (optional)

Lesson Focus

Students will learn what a rational number is, then compare and order rational numbers.

Lesson Notes

Warm-Up

- Warm-Up 1.1 on Master 1.3: Fractions to Decimals and Decimals to Fractions

Investigation: Use *Investigation Worksheet: Stock Market Report* on Master 1.7. Have students work in pairs. Circulate to help students as they work. Inform students that until very recently, North American stock markets expressed changes in stock value as rational numbers in fractional form. Ask students why the change to the decimal form of a rational number was made. (*The decimal form is easier to work with, and stock markets in the rest of the world use the decimal form*).

Discussing the Ideas: 1) No, we cannot represent a negative number as an area.
2) Any 42 shaded squares on a 100-square will represent 0.42. There are many ways to do this.
3) Yes, a positive rational number can be represented as a distance on a number line, to the right of 0.

Boggle Your Mind: Suggest the following approach to students:

- Round each number. $8\,000\,000\,000\,000 \times 2\,000\,000\,000\,000$
- Consider similar products with fewer zeros.
  $8 \times 20 = 160$, $800 \times 200 = 160\,000$, $8000 \times 2000 = 16\,000\,000$
- In each case, the number of digits in the product is the sum of the number of digits in each factor. Since there are 13 digits in each factor of the original expression, the product must have 26 digits.

Ensure students understand that the rational numbers $\frac{-3}{5}$, $\frac{3}{-5}$, and $-\frac{3}{5}$ are equivalent. The third number is the form usually used.

Exercises: For exercise 2, make copies of the 0.5-cm grid paper on Master A in the Program Overview.

For exercise 12, have students express all numbers as decimals and then as fractions with common denominators. Ask students which form of each number they preferred to work with.

For exercises 12, 14, and 16, which require a number line, insist that students use a ruler, and mark the distances between points accurately.

Communicating the Ideas: Fast food restaurant ($2.99 for a burger), movie ($8.50 for a ticket), public transit ($1.25 for a bus ride), clothing store ($19.50 for a shirt), photo store ($5.97 for film development). My bus arrives at 3:45 or a quarter to 4. I listen to 99.5 FM.

1.2 Adding and Subtracting Rational Numbers, page 18

Curriculum Correlation

Students will model problem situations involving rational numbers.

Students will add and subtract rational numbers in fractional and decimal forms, using the most appropriate method.

Suggested Materials

Masters 1.3, 1.8; scientific calculator

Lesson Focus

Students will estimate, calculate, and verify the sum and difference of rational numbers.

Lesson Notes

Warm-Up

- Warm-Up 1.2 on Master 1.3: Adding and Subtracting Integers and Decimals

Investigation: Some students will need extra practice in adding and subtracting integers before completing the Investigation. Ensure that these students complete the Necessary Skills section on Adding and Subtracting Integers at the beginning of the chapter. Encourage students to do the Investigation without calculators. Use *Investigation Worksheet: Adding and Subtracting Rational Numbers* on Master 1.8.

Discussing the Ideas: 1) A number written as +1.4 or (+1.4) is positive. A positive sign between two numbers, such as 7.1 + 1.4, represents addition. A number with no preceding sign is positive.
2) A number written as –1.4 or (–1.4) is negative. A negative sign between two numbers, such as 7.1 – 1.4, represents subtraction.

Exercises: For exercise 7, DR refers to an overdraft. Normally there would be a charge for this, but because it was for one day only, no charge was shown.

For exercise 15, remind students that a number sentence must have a relation symbol (=, <, or >) in it. Give students an example: The temperature one morning in Yellowknife is –20.2º C. It goes up 11.5º C by noon. What is the temperature at noon? (*–20.2º C + 11.5º C = –8.7º C*)

For exercise 16, point out that this is a standard way to check totals in a chart. The sum of the numbers in the bottom row should equal the sum of the numbers in the right column. This is shown in the bottom right corner of the chart.

For exercise 21, ensure students understand that when wood or metal is cut, some material is lost and the length of the remaining piece will be less than that given in the answers. Ask students to estimate how much is lost. (*the width of the saw blade*).

Communicating the Ideas: *I used the TI-30XIIS and it has a fraction key. To add* $(-\frac{3}{4}) + (-\frac{2}{3})$*, I used this keying sequence:* [(–)] *3* [$A\frac{b}{c}$] *4* [+] [(–)] *2* [$A\frac{b}{c}$] *3* [=] *to display* *–1⌟ 5/12. This represents* $-1\frac{5}{12}$.

My classmate added the same fractions, but she didn't use the fraction key, so she got a decimal answer. This is her keying sequence: [(–)] *3* [÷] *4* [=] [+] [(–)] *2* [÷] *3* [=] *to display* *–1.416666667, which is the same as* $-1\frac{5}{12}$.

1.3 Multiplying and Dividing Rational Numbers, page 24

Curriculum Correlation

Students will model problem situations involving rational numbers.

Students will multiply and divide rational numbers in fractional and decimal forms, using the most appropriate method.

Suggested Materials

Masters 1.4, 1.9; scientific calculator

Lesson Focus

Students will estimate, calculate, and verify the product and quotient of rational numbers.

Lesson Notes

Warm-Up

- Warm-Up 1.3 on Master 1.4: Multiplying and Dividing Integers

Ensure students complete the Warm-Up on Master 1.4. Skill in multiplying and dividing integers is necessary for success in this section. The Necessary Skills section at the beginning of Chapter 1, along with the extra exercises on Master 1.2, provide practice for multiplying and dividing fractions.

Investigation: Use *Investigation Worksheet: Multiplying and Dividing Rational Numbers* on Master 1.9.

Discussing the Ideas: 1a) 4.2 + 4.2 + 4.2 = 12.6; b) 3.5 + 3.5 + 3.5 + 3.5 = 14; c) Add three 4.2s and one-half of 4.2; that is $4.2 + 4.2 + 4.2 + \frac{1}{2}(4.2) = 14.7$; d) –4.2 – 4.2 – 4.2 = –12.6; e) Add three –4.2s and one-half of –4.2; that is $-4.2 -4.2 -4.2 + \frac{1}{2}(-4.2) = -14.7$; f) For (–3.5) × (–4.2), since I know that the product of two negatives is a positive, I can rewrite this as 3.5 × 4.2, which is the same as part c); 2a) $5\frac{5}{6}$ because $5\frac{5}{6} \times 6 = 35$; b) 7.2 because 7.2 × 5 = 36; c) 5.5 because 5.5 × 7.2 = 39.6; d) $-5\frac{5}{6}$ because $(-5\frac{5}{6}) \times (-6) = 35$; e) –7.2 because (–7.2) × (–5) = 36; f) 5.5 because (5.5) × (–7.2) = –39.6.

Exercises: For exercise 3, remind students that division by ½ is the same as multiplication by 2.

For exercise 16, tell students that carpet comes in rolls of a particular width, and that some stores may sell carpet only in lengths of whole numbers of metres.

For exercise 31, have students try specific values for m and n. For example, if $m = 5$ and $n = 3$, $\frac{5}{3} < \frac{4}{2}$. Have students write other examples where $m > n$ and then generalize. Ask students to examine numbers where $m = n$ and $m < n$, and draw conclusions. Ask students why $n \neq 1$. (*We cannot divide by zero*).

Communicating the Ideas: For subtracting rational numbers, perform the opposite operation (addition), and use the opposite sign of the number. (–5) – (+3) = (–5) + (–3). For dividing rational numbers, perform the opposite operation (multiplication) on the reciprocal, or opposite. $-6 \div \frac{2}{3} = -6 \times \frac{3}{2}$.

1.4 Order of Operations with Rational Numbers, page 31

Curriculum Correlation

Students will apply the order of operations in rational number computations.

Suggested Materials

Master 1.4; scientific calculator

Lesson Focus

Students will apply the correct order of operations when evaluating expressions with rational numbers.

Lesson Notes

Warm-Up

- Warm-Up 1.4 on Master 1.4: Order of Operations

Ensure students complete the Warm-Up on Master 1.4 for this section to review order of operations with integers. Circulate as students work on the warm-up to identify students who need extra practice before beginning work with rational numbers.

Some students may require extra practice with order of operations involving fractions. Before applying the order of operations to rational numbers, ask these students to evaluate expressions such as:

1. $\frac{1}{4} \times \frac{2}{3} + \frac{1}{3}$ 2. $\frac{3}{8} \div \frac{1}{4} - \frac{2}{5}$ 3. $\frac{3}{4} \times 2 - \frac{1}{3} \div \frac{5}{6}$ (*1)* $\frac{1}{2}$, *2)* $\frac{11}{10}$, *3)* $\frac{11}{10}$)

Explain to students that for contests run by companies such as McDonalds, a skill-testing question may be given to decide contest winners. Often these questions involve order of operations. Give the following example of a contest question: $4 + 3 \times 2 - 3^2$. Tell the class that four tickets were drawn before a correct answer was given. The answers given were 5, 1, 19, and 23. Ask students to determine the correct answer. (*1 is correct; if the addition is done first, the answer is 5; if –3 is squared, the answer is 19; if the addition is done first and (–3) is squared, the answer is 23*). Have students work in pairs and make up similar questions for each other.

Explain the difference between expressions such as $(-3)^2$ and -3^2. Show the expansions. $(-3)^2 = (-3)(-3) = 9$ and $-3^2 = -(3)(3) = -9$.

Discussing the Ideas: The division will be done first with or without the brackets. According to the order of operations, division should be done before addition. So, the brackets are unnecessary.

Exercises: Encourage students to complete exercises 1 to 3 without calculators and to show all steps necessary in simplifying the expressions.

Communicating the Ideas: Rules for order of operations are necessary so that expressions have unique values. Using these rules, $5 + 3 \times 2 = 5 + 6 = 11$. Without the rules, this expression might be evaluated as $8 \times 2 = 16$.

1.5 Adding and Subtracting Matrices, page 34

Curriculum Correlation

Students will model, solve, and create problems involving the matrix operations of addition and subtraction.

Suggested Materials

Masters 1.5, 1.10; scientific calculator; graphing calculator (optional)

Lesson Focus

Students will investigate the properties of matrices, and use them to solve problems in which data is added or subtracted.

Lesson Notes

Warm-Up

- Warm-Up 1.5 on Master 1.5: Operations with Decimals

For most students, the topic of matrices will be entirely new. This may provide interest. Describe alternate language for matrices:

- A 4-by-2 matrix is also called a 4×2 matrix.
- The numbers in a matrix can be called entries.

Investigation: Use *Investigation Worksheet: Adding and Subtracting Matrice* on Master 1.10. Ensure students understand that the rows in the tables represent stores, while the columns represent the number of cookie packages.

Technology Tip: If students have access to graphing calculators, you may want to have students use the Matrix function on the calculator to enter data from one or more matrices. Refer to the calculator manual for more information.

Discussing the Ideas: 1a) Yes, because A and D have the same dimensions.
b) Different because one is 2-by-3 and the other is 2-by-4.
2) The brackets define the numbers as a matrix, and isolate the numbers in the matrix from other numbers in the problem.

Exercises: For exercise 2, ask students to compare $R - S$ and $S - R$. They will find that each entry in the second matrix is the negative of the corresponding entry in the first matrix. Recognizing this will prepare students for the section on multiplying a matrix by a scalar.

For exercise 6, ensure students realize that since Price Cutter has no gummi bears, the entry for that item will be 0.

For exercise 7, students will observe that each entry for $2A$ is 2 times the entry for A. Similarly, each entry for $3A$ is 3 times the entry for A. This result will help students in understanding scalar multiplication of matrices in the next section.

Communicating the Ideas: A matrix is a rectangular display of the elements of a set. Entries in a table may be displayed in a matrix. Adding or subtracting corresponding elements of sets is easy when matrix notation is used.

1.6 Multiplying a Matrix by a Number, page 40

Curriculum Correlation

Students will model, solve, and create problems involving the matrix operation of scalar multiplication.

Suggested Materials

Masters 1.5, 1.11; scientific calculator

Lesson Focus

Students will solve problems in which data in a matrix is multiplied by a scalar.

Lesson Notes

Warm-Up

- Warm-Up 1.6 on Master 1.5: Percent

Investigation: Use *Investigation Worksheet: Multiplying a Matrix by a Number* on Master 1.11. Have students work in pairs. Circulate to help students as they work.

Inform students that matrices are usually named with upper case letters, such as *A*. Lower case letters are used for matrix entries. a_{12} refers to the entry in matrix *A* in row 1 and column 2. Stress that a 4-by-3 or 4×3 matrix always means 4 rows and 3 columns, by definition.

The matrix entries in this section are integer or decimal (dollar) amounts. Have students practice their skill with rational numbers by giving them examples such as $3\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{2}{3} & \frac{5}{9} \end{bmatrix}$ or $\frac{1}{2}\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{2}{3} & \frac{5}{9} \end{bmatrix}$ ($\begin{bmatrix} \frac{3}{2} & 1 \\ 2 & \frac{5}{3} \end{bmatrix}$ *or* $\begin{bmatrix} \frac{1}{4} & \frac{1}{6} \\ \frac{1}{3} & \frac{5}{18} \end{bmatrix}$)

Ensure students understand that adding 30% to a number is the same as multiplying the number by 1.3. For practice, have students calculate the cost of items with tax. If the tax is 15%, a \$12 t-shirt costs $\$12 + 0.15 \times \$12 = \$13.80$. The simpler way to calculate this is $1.15 \times \$12 = \$13.80$.

Discussing the Ideas: Each entry of the matrix is multiplied by the same number, so the matrix can be any size.

Exercises: For exercise 6, students may need clarification. For c), matrix *C* represents the total sales in the first quarter of next year, after the 25% increase. For d), some students may assume that 4 increases of 25% give a 100% increase, or a doubling of sales. Have students calculate the sales for the next full year by multiplying matrix *C* by 4, and they will find that her sales fall considerably short of doubling.

Communicating the Ideas: Since adding a number to itself is the same as multiplying a number by 2, $A + A = 2A$. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. $A + A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$. $2A = 2\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$.

1.7 Mathematical Modelling: How Many Cereal Boxes? page 45

Curriculum Correlation

Students will draw inferences and make predictions from a variety of displays of real-world data.

Students will represent numbers in multiple ways and apply appropriate representations to solve problems.

Suggested Materials

Master 1.6; scientific calculator; coins; grid paper; playing cards

Lesson Focus

Students model the problem of determining how many boxes of cereal must be purchased to obtain a complete set of promotional items inserted in the boxes.

Lesson Notes

Warm-Up

- Warm-Up 1.7 on Master 1.6: Finding the Mean

Each chapter in Minds on Math 9 contains one rich applied problem. *How Many Cereal Boxes* is the first of these problems that is solved by creating a mathematical model. Students will benefit from a class discussion about the problem, or by working in pairs or groups.

Ask students how many boxes they think will have to be opened if there are 2 or 4 different items in the set. Ask students what reasons they have for their answers. Ensure students realize that the problem does not have a unique solution.

Exercises: For exercises 1 to 5, ensure students understand that the more times they repeat the experiment, the more reliable their estimate will be.

For exercise 2, it will help students if you show them possible ways to record their experimental results.

For exercise 4b), suggest to students that since there are 6 possible rolls of the die, they should record a 1 or 4 as item #1, a 2 or 5 as item #2, and a 3 or 6 as item #3. For 4c), even numbers can represent item #1, and odd numbers item #2.

For exercise 5, if there are 4 items in a set, each suit can be an item. For 2 items, the red suits can be 1 item, and the blacks the other. For 13 items, use each value of the cards as one item. For 10 items, use only the cards from 1 to 10. Ensure students understand that they will have to replace each card after it is drawn, and shuffle the deck before drawing another card.

For exercise 7, ask students why they should not join the points on the graph. Joining the points is not meaningful because it is not possible to have a fractional number of items. A bar graph may be more appropriate.

Communicating the Ideas: Toss a coin several times. Count the number of tosses it takes to get at least one head and one tail. For example, if the results are T, T, T, H, it takes 4 tosses. Repeat this several times and average the results. You decide how many trials to complete.

Chapter 1 Necessary Skills

Adding and Subtracting Integers

1. Evaluate.

a) $3 + 5$ **b)** $5 - 1$ **c)** $6 - 7$ **d)** $3 - 8$

e) $-8 + 1$ **f)** $4 - 9$ **g)** $-4 - 2$ **h)** $9 - 2$

2. Evaluate.

a) $1 - 7$ **b)** $3 + 7$ **c)** $-8 + 2$ **d)** $-5 + 3$

e) $9 + 1$ **f)** $-10 + 1$ **g)** $-8 - 1$ **h)** $-2 - 5$

Multiplying and Dividing Integers

1. Multiply.

a) $(+3) \times (+5)$ **b)** $(-8) \times (+2)$ **c)** $(+6) \times (-7)$ **d)** $(-3) \times (-8)$

e) $(-8)(2)$ **f)** $(-1)(-5)$ **g)** $(0)(7)$ **h)** $(7)(-3)$

2. Divide.

a) $\frac{15}{3}$ **b)** $\frac{-15}{3}$ **c)** $\frac{15}{-3}$ **d)** $\frac{-12}{-3}$

e) $\frac{24}{-8}$ **f)** $\frac{-24}{-8}$ **g)** $\frac{-20}{5}$ **h)** $\frac{-16}{-8}$

Adding and Subtracting Fractions

1. Evaluate.

a) $\frac{1}{7} + \frac{3}{7}$ **b)** $\frac{5}{9} - \frac{2}{9}$ **c)** $\frac{4}{5} - \frac{2}{5}$ **d)** $\frac{6}{7} - \frac{2}{7}$

2. Evaluate.

a) $\frac{1}{2} + \frac{3}{7}$ **b)** $\frac{5}{6} - \frac{1}{2}$ **c)** $\frac{1}{3} + \frac{2}{3}$ **d)** $\frac{5}{8} - \frac{1}{4}$

e) $6 - \frac{1}{4}$ **f)** $\frac{2}{3} + \frac{1}{4}$ **g)** $\frac{3}{5} + \frac{1}{6}$ **h)** $\frac{7}{2} - \frac{1}{4}$

Chapter 1 Necessary Skills

Multiplying and Dividing Fractions

1. Evaluate.

a) $\frac{1}{7} \times \frac{3}{7}$ **b)** $\frac{5}{6} \div \frac{1}{2}$ **c)** $\frac{1}{3} \times \frac{2}{3}$ **d)** $\frac{5}{8} \div \frac{1}{4}$

e) $\frac{3}{7} \div \frac{3}{5}$ **f)** $\frac{1}{10} \times 5$ **g)** $6 \times \frac{2}{3}$ **h)** $\frac{3}{8} \times \frac{2}{3}$

2. Evaluate.

a) $\frac{1}{4}$ of 12 **b)** $\frac{2}{3}$ of 9 **c)** $4 \div \frac{2}{3}$ **d)** $\frac{3}{4} \div 6$

Fractions to Decimals and Decimals to Fractions

1. Write each fraction as a decimal. Use a bar for repeating decimals.

a) $\frac{1}{4}$ **b)** $\frac{2}{3}$ **c)** $\frac{2}{5}$ **d)** $\frac{7}{8}$

e) $\frac{2}{25}$ **f)** $1\frac{2}{5}$ **g)** $\frac{3}{11}$ **h)** $\frac{1}{9}$

2. Write each decimal as a fraction in lowest form.

a) 0.05 **b)** 0.4 **c)** 1.7 **d)** 3.4

e) 0.83 **f)** 0.45 **g)** 0.64 **h)** 1.25

Percent

1. Write each decimal as a percent.

a) 0.05 **b)** 0.4 **c)** 0.36 **d)** 0.82

2. Write each fraction as a percent.

a) $\frac{3}{100}$ **b)** $\frac{7}{10}$ **c)** $\frac{2}{5}$ **d)** $\frac{7}{8}$

3. Calculate. Round to the nearest tenth if necessary.

a) 30% of 50 **b)** 10% of 39 **c)** 2% of 8 **d)** 0.5% of 40

4. Write the first number as a percent of the second.

a) 3, 100 **b)** 3, 5 **c)** 8, 8 **d)** 3, 4

Warm-Up 1.1

Fractions to Decimals and Decimals to Fractions

1. Write each fraction as a decimal.

a) $\frac{1}{4}$ **b)** $\frac{3}{5}$ **c)** $\frac{7}{10}$ **d)** $\frac{3}{100}$

e) $\frac{1}{8}$ **f)** $\frac{5}{12}$ **g)** $\frac{7}{200}$ **h)** $\frac{6}{5}$

2. Write each decimal as a fraction in lowest term.

a) 0.6 **b)** 0.25 **c)** 0.71 **d)** 0.2

e) 1.2 **f)** 0.03 **g)** 0.08 **h)** 1.05

3. Write each fraction as a decimal rounded to the nearest hundredth.

a) $\frac{1}{3}$ **b)** $\frac{5}{6}$ **c)** $\frac{8}{9}$ **d)** $\frac{3}{7}$

Warm-Up 1.2

Adding and Subtracting Integers and Decimals

1. Evaluate.

a) 1.3 + 2.7 **b)** 5.8 – 1.4 **c)** 10.6 – 7.9 **d)** 8.7 + 1.4

e) 4.6 + 8.9 **f)** 12.1 – 10.3 **g)** 8.2 – 5.7 **h)** 6.5 – 5.9

2. Evaluate.

a) +5 + 1 **b)** –5 + 1 **c)** –7 – 3 **d)** 8 – 3

e) 6 + 7 **f)** –6 + 7 **g)** –7 – 8 **h)** 3 – 7

3. Evaluate.

a) 1.31 + 2.73 **b)** 5.85 – 1.41 **c)** 10.68 – 7.91 **d)** 8.70 + 1.48

e) 4.68 + 8.97 **f)** 12.15 – 10.35 **g)** 8.24 – 5.78 **h)** 6.50 – 5.95

Warm-Up 1.3

Multiplying and Dividing Integers

1. Evaluate.

a) $(-3)(+2)$ **b)** $(-5)(-2)$ **c)** $(7)(-2)$ **d)** $(-5)(-3)$

e) $(1)(-1)$ **f)** $(-1)(-1)$ **g)** $(-8)(3)$ **h)** $(7)(0)$

2. Evaluate.

a) $\frac{-20}{5}$ **b)** $\frac{-20}{-5}$ **c)** $\frac{30}{-2}$ **d)** $\frac{-40}{8}$

e) $\frac{-25}{-5}$ **f)** $\frac{-30}{6}$ **g)** $\frac{100}{-25}$ **h)** $\frac{-100}{-50}$

3. Evaluate.

a) $(2)(5)(4)$ **b)** $(-2)(5)(4)$ **c)** $(-2)(-5)(-4)$

d) $(8)(-2)(-1)$ **e)** $(-3)(-6)(2)$ **f)** $(-2)(5)(3)$

g) $(-1)(-1)(-1)$ **h)** $(-1)(-1)(-1)(-2)$ **i)** $(-1)(-1)(-1)(2)$

Warm-Up 1.4

Order of Operations

1. Evaluate.

a) $5 + 2 - 3$ **b)** $6 \times 2 + 1$ **c)** $3 + 8 \times 2$ **d)** $5 \times 3 - 2$

e) $16 - 2 \times 5$ **f)** $12 - 3 \times 4$ **g)** $20 - 8 \times 2$ **h)** $10 \times 5 - 1$

2. Evaluate.

a) $5 + 6 \div 3$ **b)** $5 - 6 \div 3$ **c)** $10 \div 5 + 1$ **d)** $10 \div 5 - 1$

e) $7 - 12 \div 3$ **f)** $20 + 20 \div 5$ **g)** $15 \div 3 + 8$ **h)** $15 \div 3 - 5$

3. Evaluate.

a) $6 \times 2 + 6 \div 2$ **b)** $8 \div 2 - 6 \div 3$

c) $10 \times 3 - 8 \div 4$ **d)** $9 \div 3 - 9 \div 9$

Warm-Up 1.5

Operations with Decimals

1. Add 3.2 to each number.

a) 7 **b)** 12 **c)** 5.1 **d)** 8.9

2. Subtract 1.4 from each number.

a) 8.4 **b)** 7 **c)** 5.9 **d)** 3.1

3. Multiply each number by 5.

a) 3.1 **b)** 2.4 **c)** 7.8 **d)** 10.1

4. Divide each number by 4.

a) 20 **b)** 2.4 **c)** 4.8 **d)** 7.2

Warm-Up 1.6

Percent

1. Write each decimal as a percent.

a) 0.2 **b)** 0.9 **c)** 0.79 **d)** 0.54

e) 0.08 **f)** 0.01 **g)** 0.001 **h)** 1.01

2. Write each fraction as a percent.

a) $\frac{1}{4}$ **b)** $\frac{1}{8}$ **c)** $\frac{3}{10}$ **d)** $\frac{4}{5}$

3. Calculate. Round to the nearest tenth if necessary.

a) 50% of 80 **b)** 50% of 8 **c)** 50% of 8.2 **d)** 25% of 64

e) 25% of 6.4 **f)** 25% of 0.64 **g)** 5% of 39 **h)** 4% of 3

Warm-Up 1.7

Finding the Mean

1. Find the mean of each group of numbers. Round answers to the nearest hundredth.

a) 2, 7, 12

b) 6, 8, 12, 3

c) 70, 70, 65, 10, 5

d) $1.50, $2.50, $11.60

2. Susan scored 59%, 65% and 80% on her geography tests. Calculate her mean test score.

3. It costs a company $1620 to prepare 5400 advertising brochures. Calculate the mean cost per brochure.

4. Between 1971 and 1999, Travis earned $1 400 000. Calculate his mean salary per year.

1.1 Investigation Worksheet: Stock Market Report

Name: _____ **Date:** _____

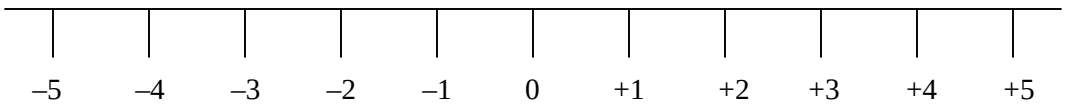
This Week on the Toronto Stock Exchange

Stock	Vol. (100s)	High	Low	Close	Chge	Last 52 wks High	Last 52 wks Low
Chin Op	14	10.10	10.10	10.10	0	12.75	8.50
CHUM	1	40.00	40.00	40.00	–5.00	56.50	36.00
Cinar rv	87	31.00	29.05	29.40	–2.50	38.25	22.70
Cinram	1086	19.85	18.75	18.75	+0.05	26.85	14.30
Cintech Tel	107	3.39	3.10	3.28	–0.11	3.89	0.60
Club Mona	1776	8.25	7.85	8.25	+0.55	14.50	3.10
Daimler	2	158.60	156.75	156.75	–0.75	164.00	128.75
Danier Leath	5	8.80	8.80	8.80	–0.20	11.95	7.00

1. Write each number in the Chge column, shown in the table below, as a fraction with numerator and denominator as integers.

Chge as a decimal	0	–5.00	–2.50	+0.05	–0.11	+0.55	–0.75	–0.20
Chge as a fraction								

2. Using the number line shown below, locate some of the numbers in the stock market report.



3. If you drew a number line showing numbers beyond 5 and beyond –5, do you think that it would be possible to locate every number in the stock market report on it? Explain.

1.2 Investigation Worksheet: Adding and Subtracting Rational Numbers

Name: _____ Date: _____

1. Look at the table below. For each sum of integers on the left, there is a sum on the right.

Integers	Sum of Integers	Rational Numbers	Sum of Rational Numbers
(+3) + (–2)		(+3.5) + (–2.1)	
(–5) + (+1)		(–5.6) + (+1.3)	
(–4) + (–3)		(–4.8) + (–3.5)	

a) In what way is the sum on the right similar to the sum on the left?

b) In what way is it different?

2. Add each pair of integers on the left and record the results in column 2. Use these results to help you add the rational numbers and then record your results in column 4.

3. Look at the table below. For each difference of integers on the left, there is a difference on the right.

Integers	Difference of Integers	Rational Numbers	Difference of Rational Numbers
(+3) – (–2)		(+3.5) – (–2.1)	
(–2) – (+7)		(–2.6) – (+7.3)	
(–6) – (–1)		(–6.8) – (–1.5)	

a) In what way is the difference on the right similar to the difference on the left?

b) In what way is it different?

4. Subtract each pair of integers on the left and record the results in column 2. Use these results to help you subtract the rational numbers and then record your results in column 4.

1.3 Investigation Worksheet: Multiplying and Dividing Rational Numbers

Name: _____ Date: _____

1. Look at the table below. For each product of integers on the left, there are two products of rational numbers on the right.

Integers	**Product of Integers**	**Rational Numbers**	**Product of Rational Numbers**	**Rational Numbers**	**Product of Rational Numbers**
(+3)(–2)		(+3.5)(–2)		(+3)(–2.5)	
(–4)(+5)		(–4.2)(+5)		(–4)(+5.1)	
(–5)(–2)		(–5.5)(–2)		(–5)(–2.2)	

a) In what ways are the products on the right similar to the one on the left?

b) In what ways are these products different?

2. Multiply each pair of integers on the left and record the results in column 2. Use these results to help you multiply the rational numbers and then record your results in columns 4 and 6.

3. Look at the table below. For each quotient of integers on the left, there are two quotients of rational numbers on the right.

Integers	**Quotient of Integers**	**Rational Numbers**	**Quotient of Rational Numbers**	**Rational Numbers**	**Quotient of Rational Numbers**
$\frac{-10}{5}$		$\frac{-9}{+5}$		$\frac{-10.5}{+5}$	
$\frac{+8}{-2}$		$\frac{+7}{-2}$		$\frac{+7.6}{-2}$	
$\frac{-6}{-3}$		$\frac{-6.3}{-3}$		$\frac{-6}{-4}$	

a) In what ways are the quotients on the right similar to the one on the left?

b) In what ways are these quotients different?

4. Divide each pair of integers on the left and record the results in column 2. Use these results to help you divide the rational numbers and then record your results in columns 4 and 6.

1.5 Investigation Worksheet: Adding and Subtracting Matrices

Name: _____ **Date:** _____

1. Draw a table to show the inventories after the stores bought the cookies.

Inventories after Stores Purchased Cookies

Store	Number of Packages of Cookies		
	Chocolate Chip	Oatmeal	Peanut Butter
Shelby's			
Price Cutters			
Food Savers			
Value Foods			

2. Draw a table to show the inventories at the end of the week.

Inventories at the End of the Week

Store	Number of Packages of Cookies		
	Chocolate Chip	Oatmeal	Peanut Butter
Shelby's			
Price Cutters			
Food Savers			
Value Foods			

1.6 Investigation Worksheet: Multiplying a Matrix by a Number

Name: _____ **Date:** _____

Amounts Charged to Stores

Store	Cost of Cookies to the Stores ($)		
	Chocolate Chip	Oatmeal	Peanut Butter
Shelby's	52.80	72.00	57.60
Price Cutters	79.20	48.00	57.60
Food Savers	52.80	72.00	115.20
Value Foods	105.60	48.00	86.40

1. Write the amounts shown in the table as matrix *B*.

2. When the stores sell the cookies to their customers they increase the prices by 30%.

a) Create matrix *I* to represent the amount by which the price increases when each store sells all the packages of each type of cookie.

b) Describe how matrix *I* is related to matrix *B*.

3. **a)** Add matrix *B* and matrix *I* to obtain a matrix that represents the amounts the stores receive when they sell all the packages of each type of cookie.

b) Find a different way to determine the matrix in part a).

Chapter 1 Test

Name: _____ **Date:** _____

Provide complete solutions to achieve full marks. If you encounter any difficulties with an exercise, leave it and return to it after you have attempted the remainder of the test.

1. Write each decimal as a fraction in simplest form.

a) 0.875 **b)** –41.6 **c)** 0.008

2. Write each fraction in decimal form. Round to 3 decimal places where necessary.

a) $-\frac{9}{100}$ **b)** $\frac{2}{7}$ **c)** $3\frac{7}{8}$

3. List in order from least to greatest.

2, $\frac{-3}{4}$, 0.7, –0.8, 0, –1, $\frac{8}{5}$

4. Evaluate.

a) 0.75 – 0.85 **b)** –0.125 – 0.35 **c)** –0.681 + 0.69

d) (0.35)(–1.2) + 6.8 **e)** –3.5 + 6.4 ÷ (–1.6) **f)** (–7.8)(–3.1) – 5.1

5. Use brackets with the expression $14 - 3 \times 6 - 5$ so that it simplifies to –9.

6. On January 18, the temperature in Gander was –18.2º C. On the same day, it was 15.4º C warmer in Halifax. What was the temperature in Halifax?

7. Evaluate.

a) $-\frac{4}{5}-\left(+\frac{1}{6}\right)$ **b)** $\left(-\frac{3}{7}\right)\left(\frac{14}{9}\right)$ **c)** $\frac{-2}{5}\div\left(\frac{-16}{25}\right)$

d) $\frac{3}{4}\times\left(\frac{1}{4}-\frac{1}{2}\right)$ **e)** $\left(\frac{2}{5}-\frac{1}{2}\right)\div\left(-\frac{1}{2}+\frac{4}{5}\right)$ **f)** $\frac{1}{4}-\left(-\frac{1}{2}\right)^2$

8. Given $x > 0$, $y > 0$, and $z < 0$, which expressions are always positive? Which are always negative?

a) $\frac{x}{y}$ **b)** $\frac{x}{z}$ **c)** $\frac{y-z}{x}$

9. Given $A = \begin{bmatrix} 3 & 2 & -8 \\ -5 & 1 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -3 \\ 6 & 1 & 7 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -2 \\ -5 & 7 \end{bmatrix}$, calculate the following.

a) $A + B$ **b)** $B - A$

c) $4C$ **d)** $A + C$

10. Alicia went to three stores to shop for new clothes.

Store	**Cost of t-shirt ($)**	**Cost of pair of jeans ($)**
Susie Style	12.95	29.95
Junior World	10.00	31.00
Ocean Tides	15.99	49.99

a) Represent this information in matrix *A*.

b) What are the dimensions of matrix *A*?

c) On a holiday weekend, all prices were reduced by 10%. Create a new matrix, *B*, showing the reduction in the price of each item.

d) Find matrix $A - B$. Explain what the resulting matrix means.

Answers to Masters

Master 1.1

Adding and Subtracting Integers

1. a)	8	**b)** 4	**c)** –1	**d)** –5
e)	–7	**f)** –5	**g)** –6	**h)** 7
2. a)	–6	**b)** 10	**c)** –6	**d)** –2
e)	10	**f)** –9	**g)** –9	**h)** –7

Multiplying and Dividing Integers

1. a)	15	**b)** –16	**c)** –42	**d)** 24
e)	–16	**f)** 5	**g)** 0	**h)** –21
2. a)	5	**b)** –5	**c)** –5	**d)** 4
e)	–3	**f)** 3	**g)** –4	**h)** 2

Adding and Subtracting Fractions

1. a)	$\frac{4}{7}$	**b)** $\frac{1}{3}$	**c)** $\frac{2}{5}$	**d)** $\frac{4}{7}$
2. a)	$\frac{13}{14}$	**b)** $\frac{1}{3}$	**c)** 1	**d)** $\frac{3}{8}$
e)	$\frac{23}{4}$	**f)** $\frac{11}{12}$	**g)** $\frac{23}{30}$	**h)** $\frac{13}{4}$

Master 1.2

Multiplying and Dividing Fractions

1. a)	$\frac{3}{49}$	**b)** $\frac{5}{3}$	**c)** $\frac{2}{9}$	**d)** $\frac{5}{2}$
e)	$\frac{5}{7}$	**f)** $\frac{1}{2}$	**g)** 4	**h)** $\frac{1}{4}$
2. a)	3	**b)** 6	**c)** 6	**d)** $\frac{1}{8}$

Fractions to Decimals and Decimals to Fractions

1. a)	0.25	**b)** $0.\overline{6}$	**c)** 0.4	**d)** 0.875
e)	0.08	**f)** 1.4	**g)** $0.\overline{27}$	**h)** $0.\overline{1}$
2. a)	$\frac{1}{20}$	**b)** $\frac{2}{5}$	**c)** $\frac{17}{10}$	**d)** $\frac{17}{5}$
e)	$\frac{83}{100}$	**f)** $\frac{9}{20}$	**g)** $\frac{16}{25}$	**h)** $\frac{5}{4}$

Percent

1. a)	5%	**b)** 40%	**c)** 36%	**d)** 82%
2. a)	3%	**b)** 70%	**c)** 40%	**d)** 87.5%
3. a)	15	**b)** 3.9	**c)** 0.2	**d)** 0.2
4. a)	3%	**b)** 60%	**c)** 100%	**d)** 75%

Master 1.3

Warm-Up 1.1

1. a)	0.25	**b)** 0.6	**c)** 0.7	**d)** 0.03
e)	0.125	**f)** $0.41\overline{6}$	**g)** 0.035	**h)** 1.2
2. a)	$\frac{3}{5}$	**b)** $\frac{1}{4}$	**c)** $\frac{71}{100}$	**d)** $\frac{1}{5}$
e)	$1\frac{1}{5}$	**f)** $\frac{3}{100}$	**g)** $\frac{2}{25}$	**h)** $1\frac{1}{20}$
3. a)	0.33	**b)** 0.83	**c)** 0.89	**d)** 0.43

Warm-Up 1.2

1. a)	4	**b)** 4.4	**c)** 2.7	**d)** 10.1
e)	13.5	**f)** 1.8	**g)** 2.5	**h)** 0.6
2. a)	6	**b)** –4	**c)** –10	**d)** 5
e)	13	**f)** 1	**g)** –15	**h)** –4
3. a)	4.04	**b)** 4.44	**c)** 2.77	**d)** 10.18
e)	13.65	**f)** 1.8	**g)** 2.46	**h)** 0.55

Master 1.4

Warm-Up 1.3

1. a)	–6	**b)** 10	**c)** –14	**d)** 15
e)	–1	**f)** 1	**g)** –24	**h)** 0
2. a)	–4	**b)** 4	**c)** –15	**d)** –5
e)	5	**f)** –5	**g)** –4	**h)** 2
3. a)	40	**b)** –40	**c)** –40	
d)	16	**e)** 36	**f)** –30	
g)	–1	**h)** 2	**i)** –2	

Warm-Up 1.4

1. a)	4	**b)** 13	**c)** 19	**d)** 13
e)	6	**f)** 0	**g)** 4	**h)** 49
2. a)	7	**b)** 3	**c)** 3	**d)** 1
e)	3	**f)** 24	**g)** 13	**h)** 0
3. a)	15	**b)** 2	**c)** 28	**d)** 2

Master 1.5

Warm-Up 1.5

1. a)	10.2	**b)** 15.2	**c)** 8.3	**d)** 12.1
2. a)	7	**b)** 5.6	**c)** 4.5	**d)** 1.7
3. a)	15.5	**b)** 12	**c)** 39	**d)** 50.5
4. a)	5	**b)** 0.6	**c)** 1.2	**d)** 1.8

Warm-Up 1.6

1. a)	20%	**b)** 90%	**c)** 79%	**d)** 54%
e)	8%	**f)** 1%	**c)** 0.1%	**d)** 101%
2. a)	25%	**b)** 12.5%	**c)** 30%	**d)** 80%
3. a)	40	**b)** 4	**c)** 4.1	**d)** 16
e)	1.6	**f)** 0.16	**g)** 1.95	**h)** 0.12

Master 1.6

Warm-Up 1.7

1. a) 7 **b)** 7.25 **c)** 44 **d)** $5.20

2. 68% **3.** $0.30 **4.** $50 000

Master 1.13

Chapter 1 Test

1. a) $\frac{7}{8}$ **b)** $-41\frac{3}{5}$ **c)** $\frac{2}{250}$

2. a) –0.09 **b)** 0.286 **c)** 3.875

3. $-1, -0.8, -\frac{3}{4}, 0, 0.7, \frac{8}{5}, 2$

4. a) –0.1 **b)** –0.475 **c)** 0.009

d) 6.38 **e)** –7.5 **f)** 19.08

5. $14 - (3 \times 6) - 5$

6. –2.8° C

7. a) $-\frac{29}{30}$ **b)** $-\frac{2}{3}$ **c)** $\frac{5}{8}$

d) $-\frac{3}{16}$ **e)** $-\frac{1}{3}$ **f)** 0

8. a) always positive **b)** always negative **c)** always positive

9. a) $\begin{bmatrix} 4 & 2 & -11 \\ 1 & 2 & 14 \end{bmatrix}$ **b)** $\begin{bmatrix} -2 & -2 & 5 \\ 11 & 0 & 0 \end{bmatrix}$ **c)** $\begin{bmatrix} 4 & -8 \\ -20 & 28 \end{bmatrix}$ d) can't be done

10. a) $\begin{bmatrix} 12.95 & 29.95 \\ 10.00 & 31.00 \\ 15.99 & 49.99 \end{bmatrix}$ **b)** 3×2 **c)** $\begin{bmatrix} 1.30 & 3.00 \\ 1.00 & 3.10 \\ 1.60 & 5.00 \end{bmatrix}$

d) $\begin{bmatrix} 11.65 & 26.95 \\ 9.00 & 27.90 \\ 14.39 & 44.99 \end{bmatrix}$

This matrix represents the prices of the items after the 10% reductions.

Relations 2

Notes for the Teacher

Chapter 2 Planning Chart

Section	Lesson Focus	Materials
Necessary Skills	Review prerequisite skills and concepts for the chapter.	Masters 2.1, 2.2; Program Masters A and C or grid paper
2.1 Investigatig Relationships in Data	Examine tables of values and graphs to identify and describe relationships between pairs of variables.	Masters 2.3, 2.6; Program Masters A and C or grid paper; string; scissors; small objects such as keys or washers; tape measures or metre sticks; masking tape; stop watch; scale that measures up to 20 kg
2.2 What Is a Relation?	Examine tables of values and graphs to identify and describe linear and parabolic relations.	Masters 2.3, 2.7; Program Masters A and C or grid paper; ruler
2.3 Graphing Relations Using an Equation	From an equation, make a table of values and graph the relation.	Masters 2.4, 2.8, 2.9; Program Masters A and C or grid paper;
2.4 Identifying Relations	Identify relations and their graphs as linear, parabolic, exponential, or none of these.	Masters 2.4, 2.10; Program Masters A and C or grid paper; ruler; assorted cans
2.5 Mathematical Modelling: Relating Animal and Human Lifetimes	Graph and compare suggested relationships between human and animal life spans.	Master 2.5; Program Masters A and C or grid paper; ruler; coloured pencils or markers (optional); overhead transparencies and markers (optional); paper clips (optional)

Scientific calculators should be available for student use throughout the chapter.

Chapter 2 Overview

Curriculum Quote

"Students should be able to move interchangeably among the various representations that describe relationships. They should describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures, and/or by problem situations. Information given in a variety of formats should be used to derive mathematical expressions and to predict unknown values."

In this chapter, students represent patterns and relationships in a variety of formats. Students learn about the different representations of patterns in relationships.

In this chapter:

- Students review how to graph data.
- Students review how to read a graph.
- Students conduct experiments to collect data and display the data in a table of values and a graph.
- Students identify relations as linear or non-linear by looking at a graph and by calculating first differences from a table of values.
- Students identify relations as parabolic by looking at a graph and by calculating second differences from a table of values.
- Students identify relations as exponential by looking at a graph and by calculating consecutive ratios of y-values from a table of values.
- Students complete tables of values from linear, quadratic, and parabolic equations, and display the values in graphical form.
- Students solve a variety of real-life problems by preparing tables of values and graphing the data.

By the end of this chapter, students should be able to:

- Collect data and complete a table of values.
- Display data given in a table on a graph.
- Identify and compare the graphs of linear and non-linear relations.
- Calculate first and second differences between values in a table.
- Calculate the consecutive ratios of y-values in a table of values.
- Classify the relation between two variables as linear, parabolic or neither of these using first and second differences.
- Classify the relation between two variables as exponential by calculating consecutive ratios of y-values from a table of values.
- Predict and justify unknown values from a graph.

Discuss the advantages and disadvantages of a mathematical model to solve a problem relating animal and human lifetimes.

Chapter 2 Necessary Skills, page 54

Suggested Materials

Masters 2.1 and 2.2; Program Masters A and C or grid paper; scientific calculator

Building Mathematical Literacy

Mathematical words and phrases for student success in Chapter 2:

coordinates	linear graph	pattern
difference	linear relations	relation
exponential graph	non-linear relations	table of values
graph	parabola	variable
horizontal axis	parabolic graph	vertical axis

Have students record unfamiliar terms in their notebooks, then use the glossary in the student book, or another reference, to research meanings. Encourage students to write definitions in their own words.

Assessment Plan

For success in Chapter 2, students need facility with the content areas addressed in the student book.

As students complete the exercises, encourage them to note areas in which they have difficulty. The chart below will help you and your students determine where they need additional practice.

Necessary Skills	Related Sections	Additional Practice
Graphing Data	2.1 to 2.5	Master 2.1
Reading a Graph	2.1 to 2.4	Master 2.1
Evaluating Expressions	2.3, 2.4	Master 2.2

Students who complete all the exercises successfully are ready to begin Section 2.1. For students having difficulty, assign additional practice as needed using the appropriate Masters listed in the table.

2.1 Investigating Relationships in Data, page 58

Curriculum Correlation

Students will represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values.

Students will interpret graphs that represent linear and non-linear data.

Students will construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity.

Suggested Materials

Masters 2.3, 2.6; Program Masters A and C or grid paper; scientific calculator; string; scissors; small objects such as keys or washers; tape measures or metre sticks; masking tape; stop watch; scale that measures up to 20 kg

Lesson Focus

Students will examine tables of values and graphs to identify and describe relationships between pairs of variables.

Lesson Notes

Warm-Up

- Warm-up 2.1 on Master 2.3: Coordinates

***Investigation*:** Use *2.1 Investigation Worksheet: A Pendulum Experiment* on Master 2.6. Have students work in pairs. When measuring the length of the pendulum, tell students to measure consistently to either the bottom *or* the top of the object. The results should be reliable, provided students take *all* measurements to the same place on the object. You may want to use a board protractor to illustrate a 30˚ angle.

Mathematical Note: Students may be interested to know that the relationship between pendulum length, l metres, and period (time for 1 swing), T seconds, is given by the formula $T = 2\pi\sqrt{\frac{l}{g}}$, where g is the acceleration due to gravity, 9.8 m/s^2. The graph of this relation is a parabola. Students will study more about the parabola in later sections of this chapter.

Remind students that when graphing from a table, the first column is usually on the horizontal axis and the second on the vertical axis. Ensure students label the axes carefully. The scales on the x- and y-axes can be different, but the numbers on a given scale must change by the same amount.

For all graphs in this chapter, Program Masters A and C provide 0.5-cm grids.

Discussing the Ideas: 1) Given one quantity, we can predict the other. 2) If the x-values do not increase by a constant amount, the y-values will not change by a constant amount, and a linear relationship can not be identified.

Exercises: Emphasize the difference between graphs with solid and dotted lines. Before beginning the exercises, have students work in pairs to discuss and decide which graphs will be solid (*4, 10, 11*) and which will be dotted (*5, 7, 8, 9*).

Ensure students understand that the graph in exercise 10 does not represent the actual motion of the spring.

For those exercises, such as exercise 10, in which students must read specific values from a graph, suggest they use a ruler placed horizontally or vertically.

Communicating the Ideas: Ensure the x-values change by a constant amount. Calculate the differences between successive y-values. If these differences are constant, the relationship is linear.

2.2 What Is a Relation?, page 67

Curriculum Correlation

Students will interpret graphs that represent linear and non-linear data.

Students will construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity.

Suggested Materials

Masters 2.3, 2.7; Program Masters A and C or grid paper; scientific calculator; ruler

Lesson Focus

Students will examine tables of values and graphs to identify and describe linear and parabolic relations.

Lesson Notes

Warm-Up

- Warm-up 2.2 on Master 2.3: Reading a Graph

Investigation: Use *2.2 Investigation Worksheet: Rectangles with the Same Perimeter* on Master 2.7. Have students work in pairs, as graphing the data may be time consuming for some. Circulate to ensure all students can draw and label the graph axes and plot the points accurately. To reinforce the idea that values for the bases of the rectangles can be numbers between the values in the given table, have students complete this table:

Base (cm)	1	1.5	2	2.5	3
Height (cm)					

Have students calculate first differences and graph the data.

Ensure students understand that for a parabolic relation, some of the first differences may be positive, and some negative.

To ensure students understand first and second differences, have them create their own table of values for a linear and a parabolic relation. Ask them to explain how they calculated the *y*-values for each table.

Discussing the Ideas: 1) For greater than 16: the same slope, but moved vertically up. For less than 16: the same slope, but moved vertically down. 2) Both are parabolic. For greater than 16: maximum *y*-value is greater. For less than 16: maximum *y*-value is less. 3a) Yes, it is non-linear because it is not a straight line. b) It appears to be parabolic, but we cannot be sure because we are only seeing positive values of *d*.

Exercises: It will be helpful for some students to draw the extended patterns in exercises 3, 4, and 5.

Communicating the Ideas: Calculate the second differences between *y*-values. A constant value means the graph is parabolic.

Extension: Have students calculate the volume of a cube with sides 1, 2, 3, 4, and 5, using the formula $V = s^3$. Graph the data. The graph may appear to be parabolic. Show students that third differences must be calculated to find a constant difference, so this graph is not a parabola. Encourage students to develop the idea that third degree equations have constant third differences. Can students guess what will be true for fourth- or fifth-degree equations?

2.3 Graphing Relations Using an Equation, page 76

Lesson Notes

Curriculum Correlation

Students will interpret graphs that represent linear and non-linear data.

Students will construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity.

Suggested Materials

Masters 2.3, 2.7; Program Masters A and C or grid paper; scientific calculator; ruler.

Warm-up:

- Warm-up 2.3 on Master 2.4: Graphing Data

Investigation 1: Use *2.3 Investigation 1 Worksheet: The Relation $y = 3x + 1$* on Master 2.8. Pair a student who has strong graphing skills with a student who has difficulties.

Investigation 2: Use *2.3 Investigation 2 Worksheet: The Relation $y = x^2 - 3x$* on Master 2.9.

Since students do not study powers until Chapter 5, some students may benefit from a review of powers such as 2^2, 2^3, 2^4, …, 3^2, 3^3, 3^4, …, and 4^2, 4^3, 4^4, … Ensure students know that $2^1$and 2 are equivalent. Remind students that the key on a scientific calculator allows them to evaluate powers. For 4^3, key in 4 3 .

Point out that any value can be used for x in a table of values for a linear or parabolic relation. Since students have not studied negative, zero, or rational exponents, the x-values for exponential graphs should be positive integers.

Students should understand that the coordinates of a point not on the graph do not satisfy the equation of the relation. Have students test this concept by selecting a point not on the graph. After substituting, they will find that the LS and RS of the equation will be different.

Discussing the Ideas: 1) Because the second differences are not constant.
2a) The second differences for consecutive y-values are constant for a parabolic relation. For an exponential relation, consecutive y-values have a constant ratio.
b) Assuming the exponential base is greater than 1, both graphs curve up, but the exponential graph increases more quickly. If the exponential base is greater than 0, but less than 1, the curve will be decreasing. c) The parabolic equation has an x-value that is always squared. The exponential equation always has x as a power of a constant.

Exercises: For exercise 5, consider introducing the idea of inverse proportionality. Ask students if they can think of other situations where one variable is inversely proportional to another (*For example: Time needed to cover a given distance is inversely proportional to the speed*). Tell students that some dermatologists say that an SPF over 15 is unnecessary. Ask students why (*Because the reduction in protection is small for each increment in SPF after 15*).

Communicating the Ideas: Calculate the ratios between consecutive y-values. If they are the same, the relation is exponential.

2.4 2.4 Identifying Relations, page 83

Lesson Focus

Students will identify relations and their graphs as linear, parabolic, exponential, or none of these.

Curriculum Correlation

Students will interpret graphs that represent linear and non-linear data.

Students will construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity.

Students will explain the connections among different representations of patterns and relationships.

Suggested Materials

Masters 2.4, 2.10, 2.11; Program Masters A and C or grid paper; scientific calculator; ruler; assorted cans

Lesson Notes

Warm-up:

- Warm-up 2.4 on Master 2.4: Classifying Relations as Linear or Non-Linear

Investigation: Use *2.4 Investigation Worksheet: Identifying Relations* on Master 2.10. Have students work in pairs.

Example 2: This example illustrates that when we model real-life situations, the resulting graphs may not be exactly linear, parabolic, or exponential, although we may use these words to describe the graphs.

Discussing the Ideas: 1a) Each value is larger than the previous. Yes, the difference would be negative if values get smaller. Negative values would result in a graph with a negative slope b) A value is multiplied by a constant to get the next value. If the ratios were less than 1, the graph would decrease as x increases. 2a) The formula for V uses an approximate value for π. b) Calculate the ratios of consecutive terms. They are not constant. c) The graph is a sketch and the scale on the vertical axis has large increments. 3) The head measurements are approximations.

Exercises: For exercise 4, students may be surprised that stopping distance is not linearly related to speed. Discuss the implications of these numbers with the class. Have students find a way to make 165.3 m a distance that students can relate to (e.g., how many 25 m swimming pools lenths?). Challenge students to calculate the approximate stopping distance for 160 km by using the second differences to find the next first difference and then the stopping distance (*approximately 218 m*).

Exercises 7 and 8 are similar. Consider asking students to complete only one of them if time is limited.

Before completing exercise 10, have students look at cans at home or in a grocery store that show both mL and fl. oz. measures. Have them bring this information to class.

Enrichment: For exercise 12, the pattern is a fractal. Ask students to investigate this subject and present their information to the class.

Communicating the Ideas: A linear graph has constant first differences (e.g., $y = 5x + 1$); a parabola has constant second differences (e.g., $y = x^2 + 7$); an exponential graph has a constant ratio for consecutive y-values. If these criteria are not true, we say the relationship is "none of these" (e.g., $y = \frac{5}{x}$).

2.5 Mathematical Modelling: Relating Animal and Human Lifetimes, page 92

Curriculum Correlation

Students will construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity.

Students will explain the connections among different representations of patterns and relationships.

Suggested Materials

Masters 2.5; Program Masters A and C or grid paper; scientific calculator; ruler; coloured pencils or markers optional: overhead transparencies; overhead markers; paper clips

Lesson Notes

Students will graph and compare suggested relationships between human and animal life spans.

Lesson Notes

Warm-Up

- Warm-Up 2.5 on Master 2.5: Making Tables of Values

Each of the three models relating animal and human lifetimes is presented with a rule. Students use a description of the relationship between animal and human lifetimes to create a table of values and a graph. After graphing each model, students answer questions about the reasonableness of the model, and how it compares with the other models.

In Model 1, make sure students understand that an increase of 1 year in a cat's age corresponds to an increase of 7 years in the person's age.

Model 2 accounts for different growth rates of cats and humans before they become adults. Ask students why they think the first year of a cat's life might equal 21 human years. This model assumes that a kitten becomes an adult cat in 1 year, while a baby takes 21 years to become an adult. Model 3 is a variation of Model 2.

***Graphing Tip*:** Since the graphs for Models 1 and 2 will appear on the same grid, have students use a different colour for each graph.

Ask students to prepare a graph of each model individually on an overhead transparency. Then you could compare the models together on the overhead projector. Give each student a copy of Master C (0.5-cm grid paper), blank transparencies, and an overhead marker. Have students use a scale of 1 square to 1 year horizontally and 1 square to 4 years vertically. To ensure that all transparencies are aligned when displayed on the overhead projector, have students use paper clips to attach the transparency to Master C as they work. When the transparencies are completed, combine them with a transparency of Master C for display on the overhead projector. Alternatively you could ask students to display their graphs on chart paper and post them on the walls around the classroom. Emphasize proper graphing techniques.

Chapter 2 Necessary Skills

Graphing Data

1. a) Copy and complete this table.

Distances Travelled at 60 km/h

Time (h)	0	3	6	9	12	15	18	21	24
Distance (km)	0	180	360						

b) Graph the data. Plot *Time (h)* along the horizontal axis and *Distance (km)* along the vertical axis.

2. Calculate the time required to travel each distance at 60 km/h. Use or extend the table or graph in exercise 1 to check.

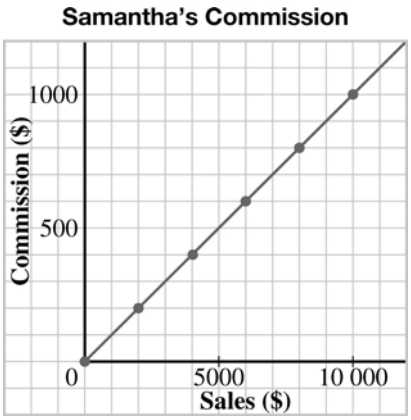
a) 120 km **b)** 600 km **c)** 1800 km **d)** 225 km **e)** 750 km

3. Calculate the distance travelled at 60 km/h in each time. Use or extend the table or graph in exercise 1 to check.

a) 4 h **b)** 2.5 h **c)** 7 h **d)** 11 h **e)** 14.5 h

Reading a Graph

This graph shows Samantha's commission for different amounts of sales.



1. Use the graph *Samantha's Commission*. Estimate her commission on each amount of sales.

a) $3000 **b)** $8000 **c)** $5000 **d)** $1000

2. Use the graph *Samantha's Commission*. What amount of sales does Samantha need to earn a commission of $700?

3. Use the graph *Population of Canada* on page 6 of your textbook.

a) Estimate the year that Canada's population was about 25 000 000.

b) Estimate Canada's population in 1994 and in 2020.

c) Estimate the increase in population from 1982 to 1996. Explain how you found this.

Chapter 2 Necessary Skills

Evaluating Expressions

1. Evaluate each expression when $n = 3$.

a) $6 - n$ **b)** $2n - 5$ **c)** $4 - 3n$ **d)** $8 + n$

e) $5n^2$ **f)** $n(n - 1)$ **g)** $1 - 4n^2$ **h)** $n^3 - n^2$

2. Evaluate each expression when $x = 2$ and x = 4.

a) 2^x **b)** 3^x **c)** 0.5^x **d)** 1.5^x

3. The circumference of a circle is given by the formula $C = 2\pi r$, where r is the radius and π is approximately 3.14. Calculate the circumference of a circle with each radius. Give answers correct to 2 decimal places.

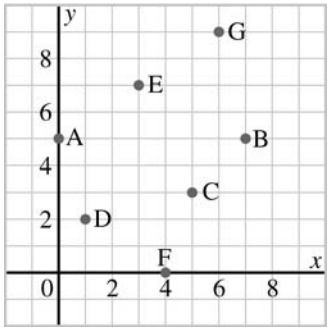
a) 10 cm **b)** 30 cm **c)** 4.2 m **d)** 10.5 m

4. The surface area of a sphere is given by the formula $A = 4\pi r^2$ where r is the radius of the sphere and π is approximately 3.14. Calculate the surface area of a sphere with each radius. Give answers correct to 2 decimal places.

Warm-Up 2.1

Coordinates

1. Write the coordinates of each point.



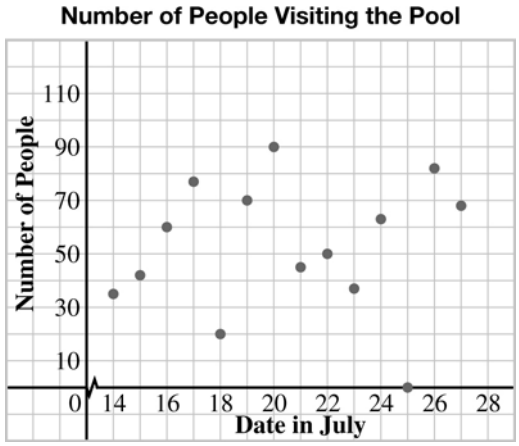
2. What are the coordinates of the origin?

3. Plot these points on a grid.
   L(4, 6), M(2, 7), N(9, 0), P(8, 1), Q(3, 3), R(0, 5), S(0, 0)

Warm-Up 2.2

Reading a Graph

1. This graph shows the number of people that visited the Long Island Aquatic Club outdoor pool each day from July 14th to July 27th.



   a) How many people were at the pool on each day?
   i) July 17th **ii)** July 23rd

   b) On what day(s) were each number of people at the pool?
   i) 20 **ii)** 45

   c) Which day had the highest attendance? The lowest attendance?

   d) Suggest reasons why the attendance was particularly high or low on the days from part c.

Warm-Up 2.3

Graphing Data

1. David went for a bike ride. The table shows his distance from home at different times.

a) Graph the data. Plot *Time (min)* along the horizontal axis and *Distance (km)* along the vertical axis. Does it make sense to join the points?

b) How far from home was David after each time?

i) 15 min **ii)** 55 min

c) After how many minutes was David each distance from home?

i) 4 km **ii)** 13 km

David's Distances from Home

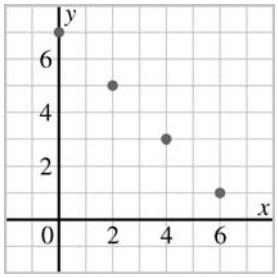
Time (min)	Distance (km)
0	0
10	2.5
20	5.0
30	7.5
40	10.0
50	12.5
60	15.0
70	17.5
80	15.0
90	12.5
100	10.0
110	7.5

Warm-Up 2.4

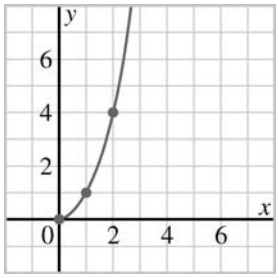
Classifying Relations as Linear or Non-Linear

1. Is each relation linear or non-linear? How do you know?

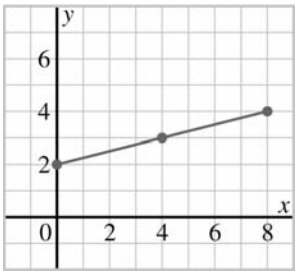
a)



b)



c)



2. Complete the difference columns, then classify each relation as linear or non-linear.

a)

x	*y*	Difference
0	5	
1	7	
2	9	
3	11	
4	13	

b)

x	*y*	Difference
0	4	
1	5	
2	7	
3	10	
4	14	

Warm-Up 2.6

Making Tables of Values

1. Complete the table of values for each situation.

a) A bean plant grows approximately 2.5 cm each day.

Time (days)	0	1	2	3	4	5	6	7	8
Height (cm)									

b) Katherine earns $5.75 for each hour worked.

Time (h)	0	1	2	3	4	5	6	7	8
Earnings ($)									

c) A car consumes 16 L of fuel every 100 km.

Distance (km)	0	100	200	300	400	500	600	700	800
Fuel Consumed (L)									

2.1 Investigation Worksheet: A Pendulum Experiment

Name:_____ **Date:**_____

Work with a partner.

You need:

- about 1 m of string
- a small object such as a key or washer
- tape
- a metre stick or measuring tape
- a watch that measures tenths of a second

1. Record your measurements from exercises 3 and 4 in this table.

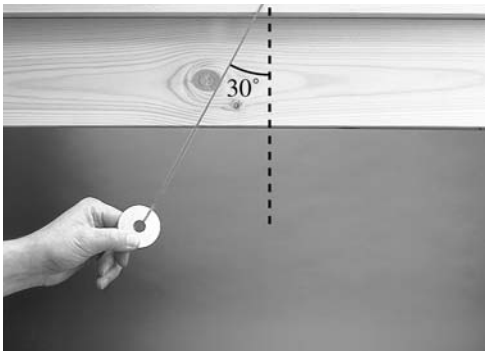
A Pendulum Experiment

Length of pendulum (cm)	Time for 5 swings (s)

2. Make a pendulum by tying an object to a string. Tape the string to the edge of a desk so the object hangs freely close to the floor.

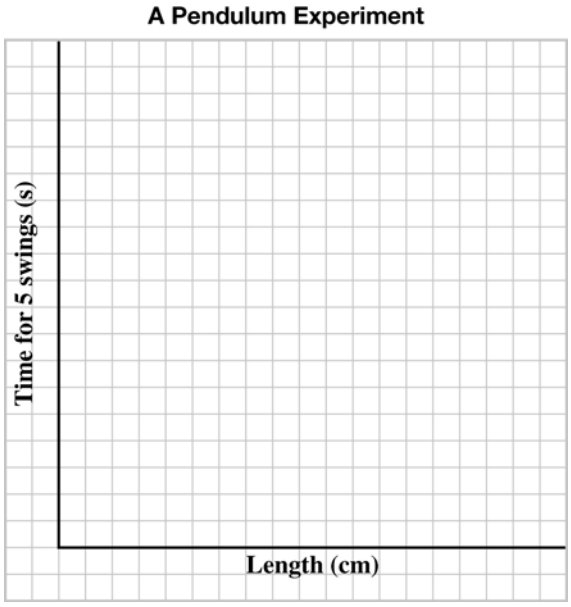
3. a) Measure the length of the pendulum. Record it in the table in exercise 1.

b) Pull the object to an angle of about 30° from the rest position. Release it. One partner can count 5 swings back and forth, while the other measures the time and records it in the table in exercise 1.



4. Repeat exercise 3 for shorter and shorter pendulum lengths. To shorten the length, lift the tape and pull up the string. Use 6 different lengths including very short lengths.

5. Graph the data, on the grid below, using appropriate scales. Decide whether to join the points.



6. a) How does the length of time change as the length of the string decreases? Explain whether this makes sense.

b) Describe the relationship between the amount of time for the pendulum to swing 5 times and the length of the string.

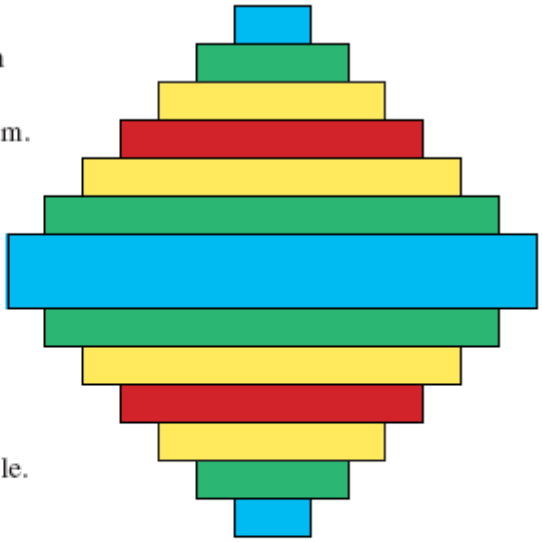
7. Use the graph to estimate how long it takes a 45-cm pendulum to swing back and forth 5 times.

8. Write a report to describe your experiment and what you found out. In your report, refer to the graph in exercise 5 and justify your findings. Attach extra sheets if necessary.

2.2 Investigation Worksheet: Rectangles with the Same Perimeter

Name:_____ **Date:**_____

This design contains 7 overlapping rectangles. There is a tall thin blue rectangle on the bottom with base 1 cm and height 7 cm. The green rectangle above it has base 2 cm and height 6 cm. The next rectangle is yellow and has base 3 cm and height 5 cm. And so on.



The blue rectangle on the top has base 7 cm and height 1 cm.

1. Measure the 7 rectangles and check that they all have perimeter 16 cm.

2. **Graphing the Height against the Base**

a) Record the bases and the heights in the table.

Base (cm)	1	2	3	4	5	6	7
Height (cm)							

b) Graph the data from the table. Plot *Base* horizontally. Does it make sense to join the points? Explain.

c) The diagram shows the rectangles with perimeter 16 cm, whose bases increase by 1 cm. Suppose more rectangles are included, whose bases increase by 0.5 cm. How would this affect the graph?

d) Visualize all possible rectangles with perimeter 16 cm. Describe how the rectangles change as the base increases from least to greatest.

3. Graphing the Area against the Base

a) Calculate the area of each rectangle. Record the results in a table.

Base (cm)	1	2	3	4	5	6	7
Height (cm)							
Area (cm^2)							

b) Graph the data from the table. Plot *Base* horizontally. Draw a smooth curve through the points.

c) Visualize all possible rectangles with perimeter 16 cm.

i) Describe how the area changes as the bases increase from least to greatest.

ii) Describe the rectangle that has the minimum area.

2.3 Investigation 1 Worksheet: Graphing Relations Using an Equation

Name:_____ **Date:**_____

1. a) Use the equation $y = 3x + 1$ to complete the table. For example, the first y-value shown is $3(-2) + 1 = -5$. Check that the second y-value is -2.

b) Determine the differences by taking each y-value and subtracting the previous y-value. For example, the difference shown is $(-2) - (-5) = -2 + 5 = 3$.

c) What do you notice about the differences?

x	y	
−2	−5	+3
−1	−2	
0		
1		
2		
3		
4		
5		

2. a) On grid paper, plot the points from your table.

b) Is the relation linear or non-linear? How do the table and the graph show this?

c) Explain why it makes sense to join the points with a straight solid line.

2.3 Investigation 2 Worksheet: The Relation $y = x^2 + 3x$

Name:_____ **Date:**_____

1. a) Use the equation $y = x^2 - 3x$ to complete the table.
For example, the first y-value shown is
$(-2)^2 - 3(-2) = 4 + 6 = 10$.
Check that the second y-value is 4.

x	y
−2	10
−1	4
0	
1	
2	
3	
4	
5	

b) Determine the differences by taking each y-value and subtracting the previous y-value. For example, the difference shown is $4 - 10 = -6$.

c) What do you notice about the differences?

2. a) On grid paper, plot the points from your table.
Draw a smooth curve through the points.
Explain why it makes sense to do this.

b) Does the relation have a parabolic graph? How do the table and the graph show this?

2.4 Investigation Worksheet: Identifying Relations

Name:_____ **Date:**_____

A hockey card is worth \$100 today. Its estimated value, v dollars, after n years is given by the equation $v = 100 \times 1.2n$.

1. Use the equation to calculate the value of the hockey card after 1 year, 2 years, 3 years, 4 years, 5 years, and 6 years. Record the results in the table.

Number of years	1	2	3	4	5	6
Value of card (\$)						

2. Graph the data from the table.

3. Does the relation between the number of years and the value of the hockey card have a linear graph, a parabolic graph, an exponential graph, or none of these? Use the table to explain your answer.

4. How would the graph change if the value of the hockey card increases by 25% each year?

Chapter 1 Test

Name _____ Date _____

Provide complete solutions to achieve full marks. If you encounter any difficulties with an exercise, leave it and return to it after you have attempted the remainder of the test.

1. Evaluate each expression for the given value.

a) $y = 4x - 1$; $x = -2$ **b)** $y = 3^x$; $x = 4$ **c)** $y = 5x^2 + 1$; $x = 4$

2. Determine whether each table represents a linear relation, a relation with a parabolic graph, an exponential relation, or none of these. Explain the reason for your choice.

a)

x	1	2	3	4	5
y	2	6	18	54	162

b)

x	1	2	3	4	5
y	–1	3	7	11	15

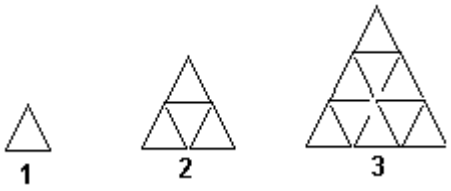
c)

x	0	1	2	3	4
y	3	6	12	21	33

3. Complete a table of values for the equation, $y = 4x^2 - 2x$.
Use $x = -2, -1, 0, 1, 2$.

x					
y					

4. Suppose this pattern is continued:



a) Complete the table to show the number of small triangles in the first 6 diagrams.

Diagram Number						
Number of Triangles						

b) Graph the data. Plot *Diagram Number* horizontally.

c) Did you use a dotted or solid line in your graph? Explain.

d) Is the relation between the diagram number and the number of triangles linear or non-linear? Explain, using the table.

e) How many triangles are in
i) diagram 7? ii) diagram 10?

f) Let d represent the diagram number, and n represent the number of triangles. Write an equation relating n and d.

5. The cost, C dollars, for a basketball team to play in a tournament is given by the formula $C = 100 + 10n$, where n is the number of players going to the tournament.

a) Complete this table of values.

Number of Players, n	5	6	7	8	9	10
Cost, C ($)						

b) Graph the relation. Plot n horizontally.

c) Is the relation linear? How do you know?

d) Use the graph to estimate how many players can go to the tournament if the team has $185.

6. The volume of liquid, V litres, poured into a container, after t hours, is given by the equation: $V = 2^t$.

a) Complete this table of values.

Time, t (hours)	0	1	2	3	4	5
Volume of Liquid, V (L)						

b) Graph the relation. Plot t horizontally.

c) Estimate the time when the tank contains 20 L.

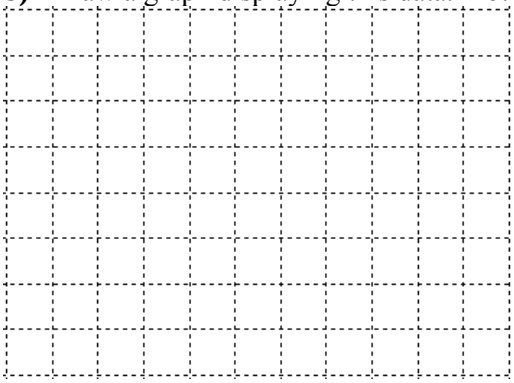
d) Does the relation have a linear graph, parabolic graph, exponential graph, or none of these? Explain.

7. Robbie is offered a summer job in telemarketing at the Atlantic A company. The job pays $25 a day plus $4 per hour worked. Another telemarketing company, Atlantic B, pays $8 per hour.

a) Complete the table of values for Atlantic A.

Number of Hours, h	1	2	3	4	5	6	7	8
Daily Pay, P								

b) Draw a graph displaying this data. Plot *h* horizontally.



c) Which company pays more for a shift of 4 hours?

d) How many hours per day would Robbie have to work at Atlantic B to earn more than he does at Atlantic A?

Answers to Masters

Master 2.1

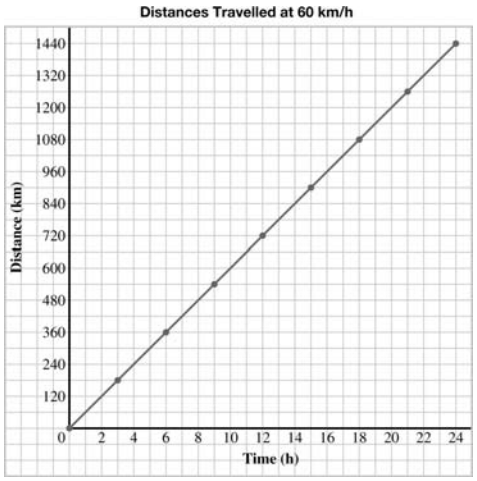
Graphing Data

1. a)

Distances Travelled at 60 km/h

Time (h)	0	3	6	9	12	15	18	21	24
Distance (km)	0	180	360	540	720	900	1080	1260	1440

b)



2. a) 2 h **b)** 10 h **c)** 30 h **d)** 3.75 h **e)** 12.5 h

3. a) 240 km **b)** 150 km **c)** 420 km **d)** 660 km **e)** 870 km

Reading a Graph

1. a) $300 **b)** $800 **c)** $500 **d)** $100

2. $7000

3. a) 1982 **b)** 29 000 000; 39 000 000 **c)** 5 000 000

Master 2.2

Evaluating Expressions

1a) 3 **b)** 1 **c)** –5 **d)** 11

e) 45 **f)** 6 g) –35 **h)** 18

2a) 4, 16 **b)** 9, 81 **c)** 0.25, 0.0625 **d)** 2.25, 5.0625

3a) 62.80 cm **b)** 188.40 cm **c)** 26.38 m **d)** 65.94 m

4a) 1256.00 cm^2 **b)** 128.61 cm^2 **c)** 35 298.04 cm^2 **d)** 1281.25 m^2

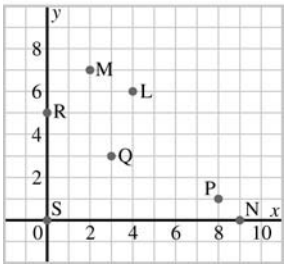
Master 2.3

Warm-Up 2.1

1. A(0, 5), B(7, 5), C(5, 3), D(1, 2), E(3, 7), F(4, 0), G(6, 9)

2. (0, 0)

3.



Warm-Up 2.2

1. a) i) 77 **ii)** 37

b) i) July 18th **ii)** July 21st

c) July 20th; July 25th

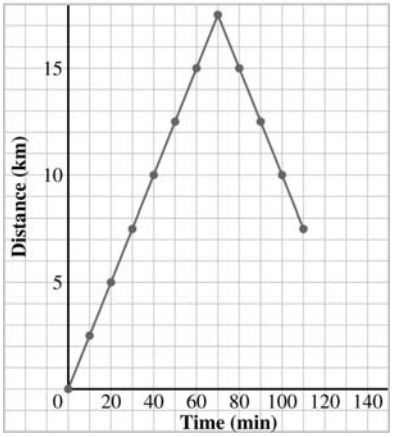
d) Answers may vary. July 20th: hot weather; July 25th: pool closed for cleaning

e) Approximately 740 people

Master 2.4

Warm-Up 2.3

1. a)



b) i) 3.75 km **ii)** 13.75 km

c) i) 16 min **ii)** 52 min, 88 min

Warm-Up 2.4

1. a) Linear **b)** Non-linear **c)** Linear

2. a)

x	y	Difference
0	5	
1	7	7 – 5 = 2
2	9	9 – 7 = 2
3	11	11 – 9 = 2
4	13	13 – 11 = 2

Linear

b)

x	y	Difference
0	4	
1	5	5 – 4 = 1
2	7	7 – 5 = 2
3	10	10 – 7 = 3
4	14	14 – 10 = 4

Non-linear

Warm-Up 2.5

a)

Time (days)	0	1	2	3	4	5	6	7	8
Height (cm)	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0

b)

Time (h)	0	1	2	3	4	5	6	7	8
Earnings ($)	0	5.75	11.50	17.25	23.00	28.75	34.50	40.25	46.00

c)

Distance (km)	0	100	200	300	400	500	600	700	800
Fuel Consumed (L)	0	16	32	48	64	80	96	112	128

Master 2.12
Chapter 2 Test

1a) –9 **b)** 81 **c)** 81

2a) Exponential because consecutive *y*-values have a common ratio of 3.

b) Linear because first differences are all 4.

c) Parabolic because second differences are all 3.

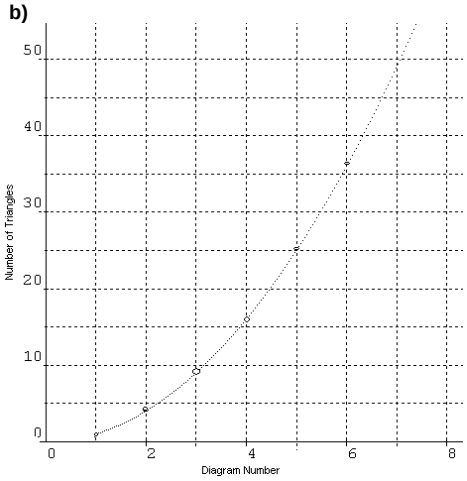
3.

x	–2	–1	0	1	2
y	20	6	0	2	12

4a)

Diagram Number, d	1	2	3	4	5	6
Number of Triangles, n	1	4	9	16	25	36

b)



c) Dotted lines because the diagonal numbers must be positive integers.

d) Non-linear because first differences are not the same.

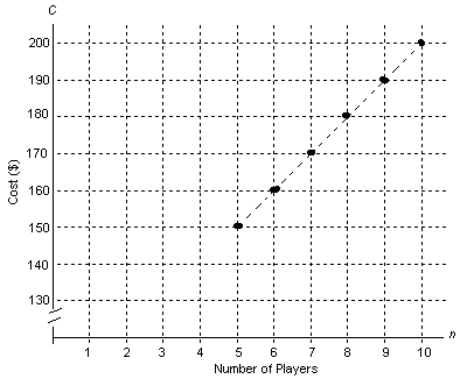
e) i) 49 ii) 100

f) $n = d^2$

5a)

Number of Players, n	5	6	7	8	9	10
Cost in Dollars, C ($)	150	160	170	180	190	200

b)



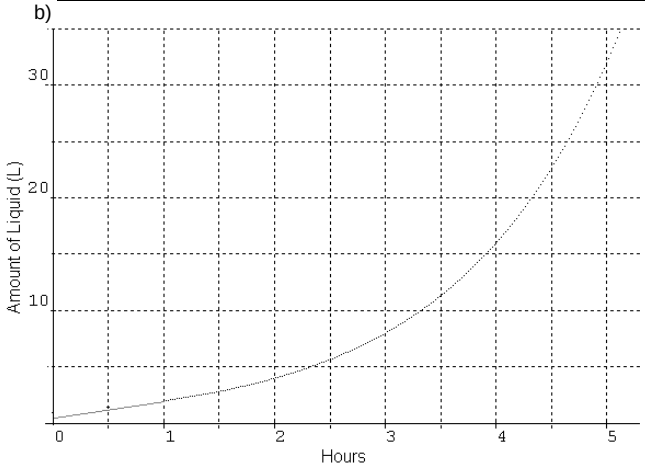
c) Linear because first differences are all 10.

d) 8 players

6. a)

Time, t (hours)	0	1	2	3	4	5
Volume of Liquid, V (L)	1	2	4	8	16	32

b)



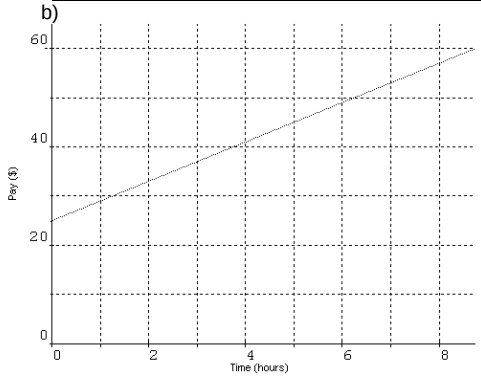
c) approximately 4.5 hours

d) Exponential because consecutive *y-values* have a common ratio of 2.

7. a)

Number of Hours, h	1	2	3	4	5	6	7	8
Daily Pay, P	29	33	37	41	45	49	53	57

b)



c) Atlantic A pays $41, Atlantic B pays $32. Atlantic A pays more

d) 7 hours

The Line 3

Notes for the Teacher

Chapter 3 Planning Chart

Section	Lesson Focus	Materials
Necessary Skills	Review prerequisite skills and concepts for the chapter.	Master 3.1; Program Masters A and C or grid paper; ruler
3.1 Slope of a Line	Graph lines, given their slopes and the coordinates of a point on each line.	Master 3.2; Program Masters A and C or grid paper; ruler
3.2 Graphing *y = mx + b*	Graph $y = mx + b$ without using a table of values, and write the equation of a line, given its slope and y-intercept.	Masters 3.2, 3.5, and 3.6; Program Masters A and C or grid paper; ruler
3.3 Applications of Linear Relationships: Part I	Given the equations of linear relations, apply the relations to solve problems involving direct and partial variation.	Master 3.3; Program Masters A and C or grid paper; ruler; computer spreadsheet (optional)
3.4 Applications of Linear Relationships: Part II	Write equations to represent linear relations and solve problems involving direct and partial variation.	Master 3.3; Program Masters A and C or grid paper; ruler
3.5 Mathematical Modelling: Setting Up for a Banquet	Use a mathematical model to estimate the number of people that can be seated in a room using different seating arrangements.	Master 3.4; Program Masters A and C or grid paper; tape measures and/or metre sticks

Scientific calculators should be available for student use throughout the chapter.

Chapter 3 Overview

Curriculum Quote

"The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the doing of mathematics. No longer is it sufficient or proper to view mathematics as a set of concepts and algorithms for the teacher to transmit to students."

In this chapter, students build on their knowledge of slopes of line segments. Linear relations are presented as equations, tables of values, graphs on grids, and descriptions of real situations.

In this chapter:

- Students use the formula $m = \frac{\text{rise}}{\text{run}}$ to calculate slopes of lines.
- Students draw a line given the slope of the line and the coordinates of a point on the line.
- Students graph lines represented by the equations $y = mx$ and $y = mx + b$.
- Students use grid paper to investigate the roles of m and b in $y = mx$ and $y = mx + b$.
- Given the slope and the coordinates of a point on a line, or the coordinates of two points on a line, students find the equation of a line.
- Students graph and interpret linear relations that represent real situations.
- Given a description of a linear relation, students write an equation to represent the relation, then graph the relation.

By the end of this chapter, students should be able to:

- Use the formula $m = \frac{\text{rise}}{\text{run}}$ to calculate slopes of lines.
- Use the slope and the coordinates of a point on a line to draw the line without using a table of values.
- Recognize $y = mx + b$ as the slope y-intercept form of the equation of a line.
- Interpret the meaning of m and b in the equations $y = mx$ and $y = mx + b$.
- Recognize $x = a$ as the equation of a vertical line and $y = b$ as the equation of a horizontal line.
- Classify linear relations as direct variation or partial variation.
- Find the equation of a line given the slope and the coordinates of a point on the line or the coordinates of two points on the line.
- Use interpolation and extrapolation to determine values of a linear relation.
- Write an equation to represent a linear relation, given a description of the relation.

Chapter 3 Necessary Skills, page 100

Suggested Materials

Master 3.1; Program Masters A and C or grid paper; ruler; scientific calculator

Building Mathematical Literacy

Mathematical words and phrases for student success in Chapter 3:

area	horizontal line	run
axes	independent variable	slope
axis	line segment	slope *y*-intercept form
collinear	linear relation	steepness
constant	ordered pair	undefined
coordinate	origin	variable
dependent variable	partial variation	varies directly
difference	perpendicular	vertical line
direct variation	relation	*x*-intercept
equation	rise	*y*-intercept

Have students record unfamiliar terms in their notebooks, then use the glossary in the student book, or another reference, to research meanings. Encourage students to write definitions in their own words.

Assessment Plan

For success in Chapter 3, students need facility with the content areas addressed in the student book.

As students complete the exercises, encourage them to note areas in which they have difficulty. The chart below will help you and your students determine where they need additional practice.

Necessary Skills	Related Sections:	Additional Practice
Slope of a Line Segment	3.1 to 3.4	Master 3.1
Linear Relations	3.1 to 3.5	Master 3.1

Students who complete all the exercises successfully are ready to begin Section 3.1. For students having difficulty, assign additional practice as needed using the appropriate Masters listed in the table.

3.1 Slope of a Line, page 102

Curriculum Correlation

Students will determine the equations of lines by obtaining their slopes and *y*-intercepts from graphs, and sketch graphs of equations using *y*-intercepts and slopes.

Suggested Materials

Master 3.2; Program Masters A and C or grid paper; ruler; scientific calculator

Lesson Focus

Students will graph lines, given their slopes and the coordinates of a point on each line.

Lesson Notes

Warm-Up

- Warm-up 3.1 on Master 3.2: Fractions

Ensure students complete the Necessary Skills exercises on Slope of a Line Segment and Linear Relations. Mastery of these concepts is essential for success in this section.

Remind students that the rise represents the vertical movement — up is positive, down is negative — and the run represents the horizontal movement — right is positive, left is negative — between points on a line. As a class, you may wish to practise breaking slopes into the rise and run. Emphasize that a positive slope indicates that the rise and run are both positive, or both negative. If the slope is negative, the rise and run have opposite signs.

Discussing the Ideas: 1a) For AB, rise is 2 and run is 3. For AD, rise is 6 and run is 9. So, for each, the fraction $\frac{\text{rise}}{\text{run}}$ is the same. b) The results in a apply for all pairs of segments in this example. For BD and CE, rises are equal and runs are equal. 2) It would not be a straight line. 3) Yes, since $\frac{1}{-3} = -\frac{1}{3}$.

Exercises: In exercises 3 to 6, students may require help to identify the run when the slope is an integer. Remind students that an integer can be written as a fraction with a denominator of 1. For exercises 3 to 14, remind students of the difference between a line segment and a line.

Exercises 6 and 7 emphasize that an infinite number of lines can be drawn with a given slope. It is probable that each student will find the coordinates of different points.

For exercise 10, it is not sufficient to merely graph the points. Students must calculate and compare the slopes of the line segments.

Communicating the Ideas: Coordinates that satisfy the equation lie on the graph of the line. Points on the graph of the line have coordinates that satisfy the equation. For $y = 3x - 2$, (1, 1) satisfies the equation and therefore lies on the graph of the line. If (1, 1) lies on the graph of the line, it satisfies the equation of the line.

Extension: A line has slope 4. It passes through A(3, 8) and B(2, *k*). What is the value of *k*? (*4*)

3.2 Graphing $y = mx + b$, page 106

Curriculum Correlation

Students will determine the equations of lines by obtaining their slopes and *y*-intercepts from graphs, and sketch graphs of equations using *y*-intercepts and slopes.

Suggested Materials

Masters 3.2, and 3.5; Program Masters A and C or grid paper; ruler; scientific calculator

Lesson Focus

Students will graph $y = mx + b$ without using a table of values, and write the equation of a line, given its slope and *y*-intercept.

Lesson Notes

Warm-Up

- Warm-up 3.2 on Master 3.2: Plotting Points on a Grid

Use *3.2 Investigation Worksheet: Using Grid Paper to Investigate $y = mx + b$* on Master 3.5. Have students label each graph with its equation.

Example 1: Students are given the equation and asked to graph the line. Emphasize that lines of the form $y = mx$ pass through the origin. Ensure students understand that the value of the *y*-intercept, or *b*, is the *y-coordinate* of the point where the line crosses the *y*-axis, not the point itself. Remind students that the slope will be positive when both rise and run are negative.

Example 2: Students are given the graph and asked to find the equation. This is the opposite of Example 1. Stress that lines going through the origin will have equations of the form $y = mx$.

Discussing the Ideas: 1a) A line through the origin, at 45˚ to both axes. b) A horizontal line, 5 units above the *x*-axis. c) A vertical line, 3 units to the left of the *y*-axis. 2) The point where a line crosses the *y*-axis has *x*-coordinate zero. The equation becomes $y = m(0) + b$ or $y = b$. The *y*-intercept is *b*. 3) Most lines can be written this way. In some equations, *m* or *b* may be zero. A vertical line is written as $x = a$ (e.g., $x = 5$), since the slope is undefined.

Exercises: Intercepts confuse some students. Stress that $y = 0$ for the *x*-intercept, and $x = 0$ for the *y*-intercept. Ask students which lines have no *x*-intercept ($y = b$, $b \neq 0$) or no *y*-intercept ($x = a$, $a \neq 0$).

For exercise 11, students can solve by:

- substituting *x* and *y* for the given point into the equation and solving for *b*.
- graphing a line through the given point with the given slope, and using the graph to find *b*. A carefully drawn graph is necessary.

A similar technique will work for exercise 12.

Communicating the Ideas: Mark the point (0, *b*) on the graph. Use the slope, written as a fraction, to show rise and run, and locate a second point on the graph. Draw a straight line through these points. The graph is a line that represents the linear relation described by the equation. For example, if $y = 3x - 4$, the *y*-intercept is –4, and the point (0, –4) is on the graph. The slope is 3 or $\frac{3}{1}$. Rise is 3 and run is 1. Rise up 3 and run right 1 from (0, –4) to find a second point on the graph.

3.3 Applications of Linear Relationships: Part I, page 113

Curriculum Correlation

Students will construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity.

Students will determine the equations of lines by obtaining their slopes and *y*-intercepts from graphs, and sketch graphs of equations using *y*-intercepts and slopes.

Students will explain the connections among different representations of patterns and relationships.

Suggested Materials

Master 3.3; Program Masters A and C or grid paper; ruler; scientific calculator; computer spreadsheet (optional)

Lesson Focus

Given the equations of linear relations, students will apply the relations to solve problems involving direct and partial variation.

Lesson Notes

Warm-up:

- Warm-up 3.3 on Master 3.3: Graphing Linear Relations

Ensure students understand that *direct variation* refers to equations of the form $y = mx$, while $y = mx + b$ is *partial variation*. *Example 1* is direct variation, while *Example 2* is partial variation.

Example 1: Emphasize the importance of choosing appropriate scales for the axes. In this example, one square represents \$1000 on the horizontal axis, and \$100 on the vertical axis. Since students may be asked to interpolate or extrapolate from graphs, they should draw graphs carefully, and allow for room to extend the graph if necessary.

Encourage students to use meaningful variable names (e.g., *a* for age, *p* for pulse, *C* for cost) for problems involving real-life situations.

Ensure students understand the distinction between independent and dependent variables. You may want to describe situations, and have students identify the independent and dependent variables.

Discussing the Ideas: 1a) Values on the axis do not begin with zero. b) The line segment would have the same slope, but would be higher up on the graph.
2) The maximum pulse rate decreases by 1 for each year a person ages.

Exercises: In exercise 6, time is the dependent variable and is plotted vertically, while in exercise 8, time is the independent variable and is plotted horizontally. For exercise 6, ensure students understand the difference between cooking time per kg (30 min), and total cooking time, *t*, in hours. Some students will need help with parts of this exercise. For e, suggest 20 min/kg so that $t = \frac{1}{3}k$. For f, suggest 40 min/kg. Ask students to find the corresponding equation ($t = \frac{2}{3}k$).

For exercise 7, students need to find the point of intersection of the graphs of the two relations. Student should then consider each company for values of *n* on either side of the point of intersection.

If students have access to computers with spreadsheet software, consider having students use the spreadsheet to find the mean cost for the situations presented in exercises 7 and 10.

Communicating the Ideas: For a linear relation, written in the form $y = mx + b$, the slope is *m*, and represents a rate of change. With a positive slope, the line goes up from left to right. With a negative slope, the line goes down from left to right. A horizontal line has zero slope.

3.4 Applications of Linear Relationships: Part II, page 120

Curriculum Correlation

Students will construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity.

Students will determine the equations of lines by obtaining their slopes and *y*-intercepts from graphs, and sketch graphs of equations using *y*-intercepts and slopes.

Students will explain the connections among different representations of patterns and relationships.

Suggested Materials

Master 3.3; Program Masters A and C or grid paper; ruler; scientific calculator

Lesson Focus

Students will write equations to represent linear relations and solve problems involving direct and partial variation.

Lesson Notes

Warm-up:

- Warm-up 3.4 on Master 3.3: Equations of Lines

In all equations, encourage students to use meaningful variables (e.g., ***t*****,** *C*, or *a*,) instead of *x* and *y*.

Example 1: Creating a table of values may help students understand the development of the equation and why the operation of multiplication makes sense in the equation. Discuss why it would be a good idea to consider a maximum number of guests.

Example 2: Compare this situation with the situation in *Example 1* and ask students to identify similarities and differences. Students should recognize that the *C*-intercept represents the fixed cost of $1000.

Discussing the Ideas: 1) Steeper slope if cost increases; less steep slope if cost decreases. 2a) The slope is the same, but the *C*-intercept moves up for an increase, or down for a decrease. b) The *y*-intercept is the same, but the slope is steeper if the coefficient increases; the slope is less steep if the coefficient decreases. 3) No, because the $25 does not include the $1000 cost to rent the hall. 4) In both examples, the slope is 25. In *Example 1*, this is the cost per person, and hence the change for each additional person. In *Example 2*, $25 is not the actual cost per person, but is the change in the total cost for each additional person.

Exercises: For exercises 7, 8, 9, and 11, have students include the units with the rise and run. This will help them determine the units for the slope. Some students may find a table of values helpful in completing part f of exercises 7 and 8.

For exercise 10, you may need to tell students that the break-even point occurs when revenue equals cost. In other words, students need to determine the value of *t* at the point of intersection of the two lines.

Communicating the Ideas: 1) Two points. For example, given (1, 3) and (2, –4); then $m = -7$ and the equation is $y = -7x + 10$. 2) A point and a slope. For example, given (1, 3) and $m = 4$; then $y = 4x - 1$.

3.5 Mathematical Modelling: Setting Up for a Banquet, page 126

Curriculum Correlation

Students will construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity.

Students will explain the connections among different representations of patterns and relationships.

Suggested Materials

Master 3.4; Program Masters A and C or grid paper; tape measures and/or metre sticks; scientific calculator

Lesson Focus

Students will use a mathematical model to estimate the number of people that can be seated in a room using different seating arrangements.

Lesson Notes

Warm-up:

- Warm-up 3.5 on Master 3.4: Linear Relations

Remind students that a mathematical model is a way to solve an applied problem. In this model, the problem requires estimating the number of people to be seated in a room for different arrangements of chairs and/or tables. Have students talk about the seating arrangements in places they have been. Have some students measure the school cafeteria, count the number of tables, and calculate whether the rules suggested in the model have been followed.

Students may wish to look up the word *oblong* in the dictionary to find that this word refers to a rectangle that is not a square.

Imperial units are used in this section because the company that rents the equipment uses them. Students will find that most businesses that do home repair and home decorating use Imperial measurements as this is the U.S. market standard. Ensure students know that one foot as a unit of measurement is approximately 30.48 cm. Some rulers are 30 cm long and this can be used to estimate one foot. Inform students that 1 foot = 12 inches, and 3 feet = 1 yard.

Exercises: For exercises 4, 6, and 8, all equations have the form $y = mx$, although students may not recognize this. Remind students that $y = \frac{x}{6}$ can also be written as $y = \frac{1}{6}x$. Ensure students recognize that as the denominator increases, the slope decreases.

For exercises 5, 7, and 9, students should use both the equation and the graph to estimate the number of people who can be seated.

Communicating the Ideas: Find the area of the banquet room in square feet. Decide on the type of seating to be used. Divide the area by 6 for theatre seating, by 8 for oblong tables, and 10 for round tables. This gives the approximate number of people who can be seated.

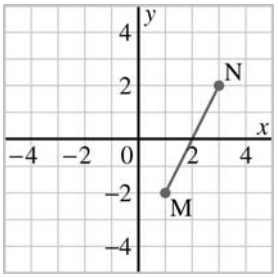
Extension: According to the rental company, these models are for maximum seating. The company's brochure states that if space is available for more comfortable seating, 2 additional square feet per person should be allowed. Students could determine how the previous results would be affected if more comfortable seating is desired. Note that $y = \frac{1}{8}x$, $y = \frac{1}{10}x$, and $y = \frac{1}{12}x$ are the equations for the theatre style, oblong table, and round table models.

Chapter 3 Necessary Skills

Slope of a Line Segment

1. Determine the slope of each line segment.

a)



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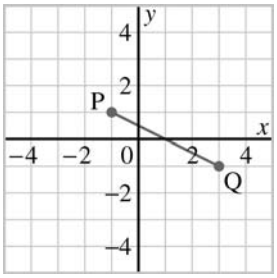

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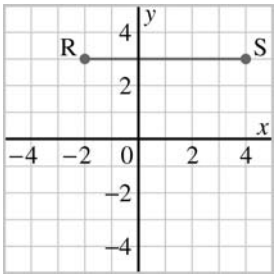


2. Graph each line segment. Find its slope.

a) A(3, –2), B(–1, 2) **b)** C(–8, 1), D(–6, 5) **c)** E(–5, 1), F(0, –1)

d) G(–6, –5), H(–5, –1) **e)** I(2, 2), J(7, 5) **f)** K(7, –5), L(–1, –2)

3. a) What is the slope of the line segment with endpoints X(2, 3) and Y(5, 3)?

b) What is the slope of the line segment with endpoints M(–4, 6) and N(–4, 9)?

Linear Relations

1. a) Copy and complete the table of values for each relation.

i) $y = 2 - 5x$

x	y	Difference
–1		
0		
1		
2		
3		

ii) $y = 3x + 2$

x	y	Difference
–2		
–1		
0		
1		
2		

b) Graph each relation.

c) What is the slope of any segment of the graph?

d) What do you notice about the slope of the graph and the numbers in the Difference column?

2. For each equation in exercise 1:

a) Is the relation linear?

b) Explain how the table of values shows whether the relation is linear.

c) Explain how the slope shows whether the relation is linear.

Warm-Up 3.1

Fractions

1. Reduce to lowest terms.

a) $\frac{15}{25}$ **b)** $\frac{42}{63}$ **c)** $\frac{-18}{24}$ **d)** $\frac{-44}{-100}$

2. In each case, determine if the fractions are equivalent.

a) $\frac{20}{12}, \frac{-15}{-9}, \frac{75}{45}$ **b)** $\frac{-3}{8}, \frac{36}{-96}, \frac{9}{-24}$ **c)** $\frac{32}{40}, \frac{40}{-50}, \frac{12}{15}$

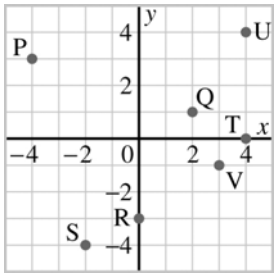
3. Order the fractions from least to greatest.

$\frac{-7}{12}, \frac{3}{4}, \frac{-4}{7}, \frac{1}{3}, \frac{5}{9}, \frac{3}{11}$

Warm-Up 3.2

Plotting Points on a Grid

1. Name the coordinates of each point.



2. Plot these points on a coordinate grid.

A(0, –5), B(–6, 1), C(3, 6), D(1, –1), E(–5, –3), F(–1, 0), G(–2, –6), H(–4, 5), I(6, –5), J(6, 1)

Warm-Up 3.3

Graphing Linear Relations

1. Complete each table of values, then graph each line on the same grid.

a) $y = 3x + 2$ **b)** $y = 2x - 1$ **c)** $y = -\frac{2}{3}x + 1$

x	y
–2	
–1	
0	
1	

x	y
–1	
0	
1	
2	

x	y
–3	
0	
3	
6	

2. Determine the slope and y-intercept for each line, then graph each line on the same grid.

a) $y = -x + 4$ **b)** $y = \frac{1}{2}x - 3$ **c)** $y = 5x$

3. Graph each line on the same grid.

a) $y = -3$ **b)** $x = 1$ **c)** $y = 4$

Warm-Up 3.4

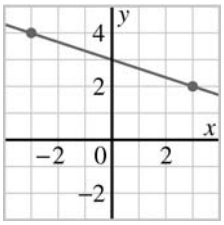
Equations of Lines

1. Write the equation of the line with each slope and y-intercept.

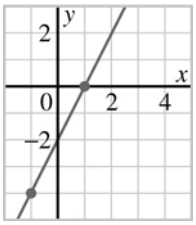
a) $m = 7, b = -2$ **b)** $m = -\frac{4}{3}, b = 1$ **c)** $m = 0, b = 4$

2. Find the equation of each line.

a)



b)



3. Find the equation of the line that passes through A(5, 4) and B(–5, –2).

Warm-Up 3.5

Linear Relations

1. Complete each table of values, then graph each relation.

a) $C = 250 + 15n$

n	C
0	
1	
2	
3	

b) $d = 65t$

t	d
0	
1	
2	
3	

c) $p = 0.2c + 30$

c	p
0	
10	
20	
30	

2. Identify each relation in exercise 1 as direct or partial variation. Explain your choice.

3.2 Investigation Worksheet: Using Grid Paper to Investigate $y = mx$

Name:_____ **Date:**_____

Equations of the form $y = mx$

1a). Complete the tables of values for each equation.

$y = x$

x	y
–2	
0	
2	

$y = 2x$

x	y
–2	
0	
2	

$y = \frac{1}{2}x$

x	y
–2	
0	
2	

$y = -x$

x	y
–2	
0	
2	

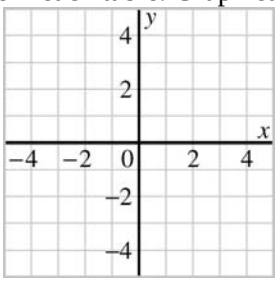
$y = -2x$

x	y
–2	
0	
2	

$y = -\frac{1}{2}x$

x	y
–2	
0	
2	

b) Use the coordinates from each table. Graph each equation in exercise 1 on the grid below. Label each line.



c) How are the graphs the same? How are they different?

2a) Find the slope of each line in exercise 1.

b) Compare the slope of each line with its equation. What do you notice?

3) Each equation in exercise 1 is in $y = mx$ form. What do you think m represents?

Equations of the Form $y = mx + b$

4a) Complete the table of values for each equation.

i) $y = x + 5$

x	y
–2	
0	
2	

$y = -\frac{1}{2}x + 5$

x	y
–2	
0	
2	

$y = \frac{1}{2}x + 5$

x	y
–2	
0	
2	

$y = –x + 5$

x	y
–2	
0	
2	

ii) $y = 2x + 3$

x	y
–2	
0	
2	

$y = 2x – 1$

x	y
–2	
0	
2	

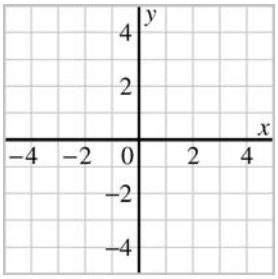
$y = 2x + 1$

x	y
–2	
0	
2	

$y = 2x – 3$

x	y
–2	
0	
2	

b)Graph all six equations on the grid at the right. Label each line.



c) How are the graphs in part a the same? How are they different?

5a) Determine the slope of each line in exercise 4.

b) Compare each slope with the corresponding equation. What do you notice?

6) Each equation you graphed in exercise 4 has the form $y = mx + b$. Compare the equation and the graph of each line. What does b represent?

Chapter 3 Test

Name:_____ **Date:**_____

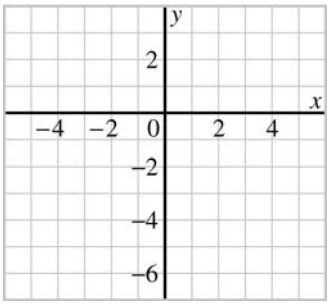
Provide complete solutions to achieve full marks. If you encounter any difficulties with an exercise, leave it and return to it after you have attempted the remaindert of the test.

1. Determine the slope and y-intercept of each line.

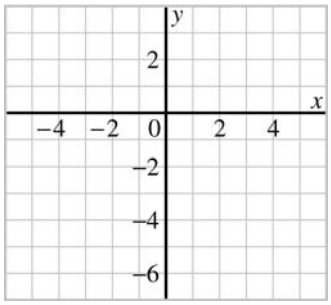
a) $y = 6x$ **b)** $y = -5x + 3$ **c)** $x = 5$

2. Graph the line represented by each equation.

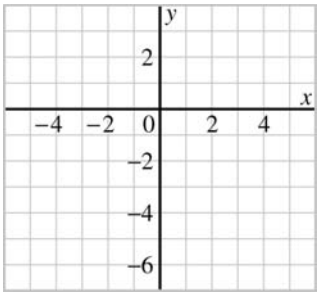
a) $y = \frac{3}{5}x - 2$



b) $y = 3x$



c) $x = -1$



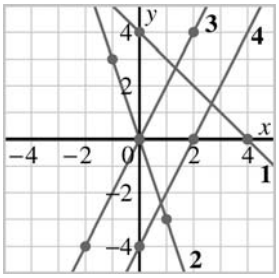
3. Match each line with its corresponding equation. Write your answer in the space provided.

a) $y = 2x$ _____

b) $y = -x + 4$ _____

c) $y = -3x$ _____

d) $y = 2x - 4$ _____



4. a) Explain how you can determine whether the points A(1, 4), B(–2, –5), and C (4, 12) are collinear.

b) Determine whether the points A, B, and C, are collinear.

c) Find another point that is collinear with points A and B.

5. Determine the equation of each line.

a) The line with slope $\frac{4}{5}$ and y-intercept –3

b) The line with slope –5 that passes through the origin

c) The vertical line that passes through the point (2, 4)

d) The x-axis

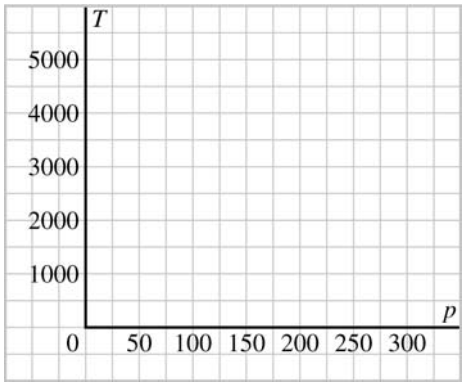
6. The point (–6, 2) lies on the line $y = -\frac{2}{3}x + b$.

Determine the value of b. Explain how you got your answer.

7. The cost of organizing a boat cruise is $500 plus $15 for each person that attends. Let T represent the total cost. Let p represent the number of people that attend.

a) Write an equation relating T and p.

b) Graph the relation.



c) Find the slope. What does the slope represent?

d) Find the T-intercept. What does the T-intercept represent?

e) Use the graph to estimate the cost if 36 people attend.

f) Use the equation to calculate the cost if 36 people attend.

g) Suppose the cost was $3 more per person attending. Describe how the equation and graph would change.

h) Suppose the fixed cost increased to $650. Describe how the equation and graph would change.

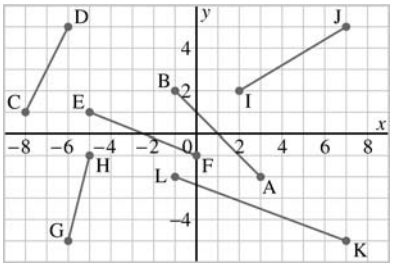
Answers to Masters

Master 3.1

Slope of a Line Segment

1. a) 2 **b)** $\frac{-1}{2}$ **c)** 0

2.



a) –1 **b)** 2 **c)** $-\frac{2}{5}$ **d)** 4 **e)** $\frac{3}{5}$ **f)** $-\frac{3}{8}$

3. a) 0 **b)** undefined

Linear Relations

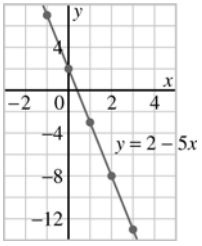
1. a) i)

x	y	Difference
–1	7	
0	2	–5
1	–3	–5
2	–8	–5
3	–13	–5

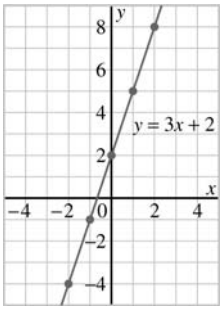
ii)

x	y	Difference
–2	–4	
–1	–1	3
0	2	3
1	5	3
2	8	3

b) i)



ii)



c) i) –5 **ii)** 3

d) i) They are equal. **ii)** They are equal.

2. a) i) Yes **ii)** Yes

b) i), ii) The differences between consecutive *y*-values are equal.

c) i), ii) The slopes of all the segments of the line are equal.

Master 3.2

Warm-Up 3.1

1. a) $\frac{3}{5}$ **b)** $\frac{2}{3}$ **c)** $-\frac{3}{4}$ **d)** $\frac{11}{25}$

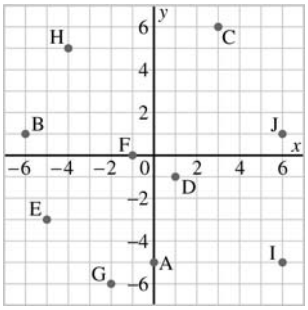
2. a) Yes **b)** Yes **c)** No

3. $\frac{-7}{12}, \frac{-4}{7}, \frac{3}{11}, \frac{1}{3}, \frac{5}{9}, \frac{3}{4}$

Warm-Up 3.2

1. P(–4, 3), Q(2, 1), R(0, –3), S(–2, –4), T(4, 0), U(4, 4), V(3, –1)

2.



Master 3.3

Warm-Up 3.3

1. a)

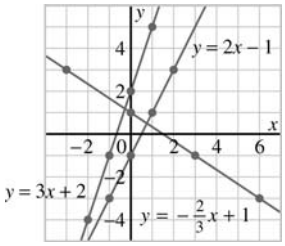
x	y
–2	–4
–1	–1
0	2
1	5

b)

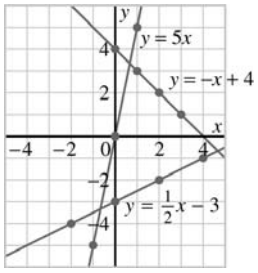
x	y
–1	–3
0	–1
1	1
2	3

c)

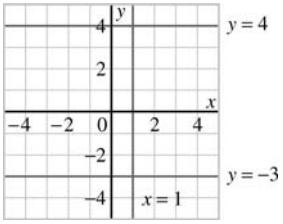
x	y
–3	3
0	1
3	–1
6	–3



2. a) –1; 4 **b)** $\frac{1}{2}$; –3 **c)** 5; 0



3.



Warm-Up 3.4

1. a) $y = 7x - 2$ **b)** $y = -\frac{4}{3}x + 1$ **c)** $y = 4$

2. a) $y = -\frac{1}{3}x + 3$ **b)** $y = 2x - 2$

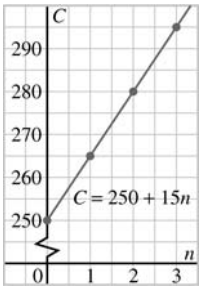
3. $y = \frac{3}{5}x + 1$

Master 3.4

Warm-Up 3.5

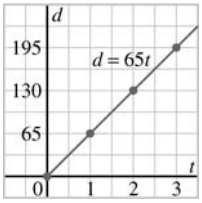
1. a)

n	C
0	250
1	265
2	280
3	295



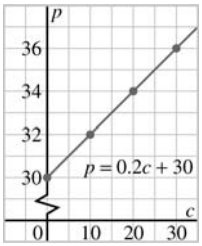
b)

t	d
0	0
1	65
2	130
3	195



c)

c	p
0	30
10	32
20	34
30	36



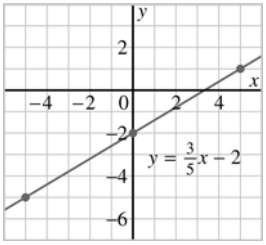
2. a) Partial variation **b)** Direct variation **c)** Partial variation

Master 3.7

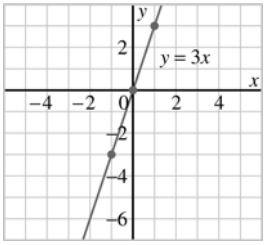
Chapter 3 Test

1. a) 6; 0 **b)** –5; 3 **c)** Undefined; no *y*-intercept

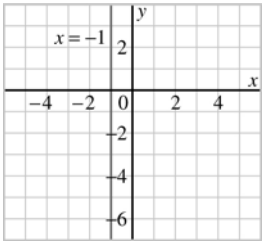
2. a)



b)



c)



3. a) 3 **b)** 1 **c)** 2 **d)** 4

4. a) Explanations may vary. The line segments AB ad BC have a common point, B. Check to see if AB=slope BC. If the slopes are the same, the points are collinear.

b) Slope AB = 3, slope BC = $\frac{17}{6}$, slope. The points are not collinear.

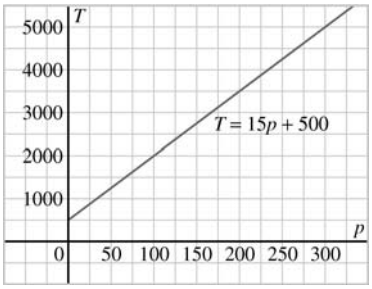
c) Answers may vary. Choose the point D(4, 13). Then slope AB=slope BD = 3.

5. a) $y = \frac{4}{5}x - 3$ **b)** $y = -5x$ **c)** $x = 2$ **d)** $y = 0$

6. $b = -2$

7. a) $T = 15p + 500$

b)



c) The slope is 15. It represents the additional cost per person.

d) The T-intercept is 500. It represents the fixed cost of $500.

e) Answers may vary; Approximately $1000

f) $1040

g) The equation would be $T = 18p + 500$. The graph would be steeper.

h) The equation would be $T = 15p + 650$. The graph would shift up.

Congruence and Similarity

4

Notes for the Teacher

Chapter 4 Planning Chart

Section	Lesson Focus	Materials
Necessary Skills	Review prerequisite skills and concepts for the chapter.	Master 4.1
4.1 Constructing Triangles Using Ruler and Protractor	Construct triangles given the measures of two angles and the contained side, or two sides and the contained angle.	Masters 4.2, 4.6; ruler; protractor; magazines and newspapers; overhead transparencies and markers (optional)
4.2 Congruent Line Segments, Angles, and Figures	Identify congruent line segments, angles, and figures.	Masters 4.2, 4.7, 4.8; paper; scissors; tracing paper; protractor; ruler
4.3 Conditions for Congruence	Describe the information required to determine whether two triangles are congruent.	Masters 4.3, 4.9, 4.10; ruler; protractor; compasses
4.4 Similar Figures	Apply proportions to describe and solve similar figures.	Masters 4.3, 4.11; ruler; protractor
4.5 Conditions for Similarity	Identify similar triangles and find unknown measures in pairs of similar triangles.	Masters 4.4, 4.12, 4.13; ruler; protractor
4.6 Solving Problems Using Similar Triangles	Use properties of similar triangles to find unknown distances in applied situations.	Master 4.4; ruler
4.7 Mathematical Modelling: Solar Eclipses	Use similar triangles to model a total eclipse of the sun.	Master 4.5; paper; measuring tape or metre sticks; compasses; tape; scissors
Chapter Test	Consolidate and assess chapter content.	Master 4.14

Scientific calculators should be available for student use throughout the chapter.

Chapter 4 Overview

Curriculum Quote

"There are many possibilities for integrated learning experiences — through learning centres, teacher-directed activities, group or independent exploration, and other opportune learning situations. However, it should be remembered that certain aspects of mathematics are sequential, and need to be developed in the context of structured learning experiences."

In this chapter, students use hands-on activities to explore the properties of congruent and similar triangles. The work in this chapter will particularly appeal to visual learners and those who learn best through hands-on experiences.

In this chapter:

- Students learn the vocabulary of angles and triangles.
- Students construct triangles using a ruler and a protractor.
- Students determine the conditions that will produce unique triangles.
- Students explore the properties of congruent triangles.
- Students determine the conditions that will guarantee congruent triangles.
- Students use congruent triangles and angle properties to make informal deductions.
- Students determine the conditions that will guarantee similar figures.
- Students apply their skill in using proportions to solve problems involving similar triangles.
- Students relate congruence and similarity of triangles.

By the end of this chapter, students should be able to:

- Explain and give examples for the vocabulary of angles and triangles.
- Use a ruler and a protractor to construct triangles satisfying given conditions.
- Construct congruent figures using paper folding.
- Identify congruent figures.
- Use a ruler, a protractor, and compasses to construct congruent triangles.
- Give reasons why given triangles are congruent, or not congruent.
- Calculate the equal sides and angles in congruent triangles.
- Identify similar figures.
- Give reasons why given triangles are similar, or not similar.
- Determine the measure of corresponding sides and angles of similar figures.
- Solve real-life problems using similar triangles.

Chapter 4 Necessary Skills, page 136

Suggested Materials

Master 4.1; scientific calculator

Building Mathematical Literacy

Mathematical words and phrases for student success in Chapter 4:

acute angle
acute triangle
alternate angles
angle
complementary angles
congruent
corresponding angles
diagonal
equiangular
equilateral triangle
geometry
interior angles
isosceles triangle
line segment
median
obtuse angle
obtuse triangle
opposite angles
parallel lines
point
proportion
protractor
quadrilateral
ratio
right angle
right triangle
ruler
scale
scalene triangle
similarity
straight angle
supplementary angle
vertex

Have students record unfamiliar terms in their notebooks, then use the glossary in the student book, or another reference, to research meanings. Encourage students to write definitions in their own words.

Assessment Plan

For success in Chapter 4, students need facility with the content areas addressed in the student book.

As students complete the exercises, encourage them to note areas in which they have difficulty. The chart below will help you and your students determine where they need additional practice.

Necessary Skills	Related Sections	Additional Practice
Parallel Lines, Triangles, and Related Angles	4.2, 4.3, 4.5, 4.6	Master 4.1
Ratio and Proportion	4.4 to 4.7	Master 4.1

Students who complete all the exercises successfully are ready to begin Section 4.1. For students having difficulty, assign additional practice as needed using the appropriate Masters listed in the table.

4.1 Constructing Triangles Using Ruler and Protractor, page 139

Curriculum Correlation

Students will be expected to investigate, and demonstrate an understanding of, the minimum sufficient conditions to produce unique triangles.

Suggested Materials

Masters 4.2, 4.6; ruler; protractor; magazines and newspapers; overhead transparencies and markers (optional); scientific calculator

Lesson Focus

Students will construct triangles given the measures of two angles and the contained side, or two sides and the contained angle.

Lesson Notes

Warm-Up

- Warm-Up 4.1: Equations and Percent on Master 4.2

Ensure all students have protractors. For those students who lose or forget their protractors, you can create an extra supply by photocopying a protractor several times on one page to create a master sheet. Photocopy the master onto overhead transparency sheets, then cut out the protractors.

Investigation 1: Have students work individually or with partners. When students compare their triangles with those of other students, all triangles should look the same. If they do not, students may be using their protractors incorrectly. With this investigation, as well as the others, you may want to prepare an overhead transparency showing all triangles for student comparison. Preparing these transparencies could be a worthwhile assignment for capable students.

Investigation 3: Use *4.1 Investigation 3 Worksheet: Constructing Triangles* on Master 4.6. Encourage students to make sketches before they start their constructions.

Discussing the Ideas: Measure the angles of each triangle. If any angle is greater than 90º, the triangle is obtuse. If any angle is equal to 90º, the triangle is right. If all angles are less than 90°, the triangle is acute.

Exercises: In exercise 1, check that students understand the angle symbols used in the diagrams (the small arc, the small square-corner, and the small semicircle). Explain that each pair of line segments forms two angles, and the symbol indicates which angle to consider.

For exercise 4, students will see the different triangles more easily if they use a different colour for each.

For exercises 10 and 12, students may recall that the sum of the measures of the angles of a triangle is always 180º. For an equilateral triangle, each angle is 180º ÷ 3, or 60º, and therefore none of the angles will be obtuse or right.

Communicating the Ideas: Provide students with magazines and newspapers if you think they may not have access to them at home. Home decor magazines or the home section of newspapers provide pictures, floor plans, and advertisements that incorporate triangles into designs.

4.2 Congruent Line Segments, Angles, and Figures, page 146

Curriculum Correlation

Students will be expected to make informal deductions using congruent triangle and angle properties.

Suggested Materials

Masters 4.2, 4.7, 4.8; paper; scissors; tracing paper; protractor; ruler; scientific calculator

Lesson Focus

Students will identify congruent line segments, angles, and figures.

Lesson Notes

Warm-Up

- Warm-Up 4.2: Angles on Master 4.2

Investigation 1: Ensure students understand that the paper folds are all parallel. Consider giving students coloured paper or wrapping paper, so that their congruent figures can be displayed. Encourage students to be creative when drawing the figure on the fold line.

Investigation 2: Use *4.2 Investigation 2 Worksheet: Congruent Line Segments and Angles* on Master 4.7. When checking whether angles are congruent, students should recognize that the lengths of the rays are irrelevant.

Investigation 3: Use *4.2 Investigation 3 Worksheet: Opposite Angles* on Master 4.8. This investigation should lead students to the conclusion that opposite angles are equal.

Discussing the Ideas: **1a)** The lengths of AB and CD are the same. **b)** The measures of the angles are the same. **2a)** So that they can be used in, and counted by, machines **b)** So that a lock will accept only one size and shape of key **c)** To fit into dispensing machines, and so that they contain standard amounts of liquid **d)** So that the main element of the game is constant

Exercises: For exercise 7, students can use tracing paper, rulers, or protractors to determine congruence. The exact conditions for congruence will be established in the next section.

For exercise 8, encourage students to find the matching pair by looking at the diagrams. Then, have students measure with a ruler or protractor to confirm their conclusions.

For exercise 13, suggest to students that there are more triangles than they may see at first glance. Have students systematically analyze each figure. For example, in part a, how many triangles can they find that include segment AB as one side? (ΔABC, ΔABD, ΔABE, *and* ΔABF)

Exercise 16 is important in preparing students to find the conditions necessary for congruence.

Communicating the Ideas: Congruence means that things are identical in size and shape. For line segments, congruence means equal lengths. For angles, congruence means equal measure. Congruent triangles have equal angles and sides of equal length.

4.3 Conditions for Congruence, page 152

Curriculum Correlation

Students will be expected to make informal deductions using congruent triangle and angle properties.

Students will be expected to investigate, and demonstrate an understanding of, the minimum sufficient conditions to produce unique triangles.

Students will be expected to investigate, and demonstrate an understanding of, the properties of and the minimum sufficient conditions to guarantee congruent triangles.

Suggested Materials

Masters 4.3, 4.9, 4.10; ruler; protractor; compasses; scientific calculator

Lesson Focus

Students will describe the information required to determine whether two triangles are congruent.

Lesson Notes

Warm-Up

- Warm-Up 4.3: Determining Angle Measures on Master 4.3

Students use compasses in this section. Good compasses will have a screw that locks the compasses into a fixed position. Demonstrate the use of compasses on the blackboard. Show students how to rotate the point leg of the compasses using the thumb and index finger, allowing the pencil to move smoothly around the centre point. Ensure students use a sharp pencil. For students needing help with the use of compasses, assign exercises such as the following:

- Construct three or more different-sized circles. Ensure that the radius of the compasses remains constant while drawing each circle.
- With a ruler, draw 4-cm and 6-cm line segments. Construct a circle with the compasses set at each length.

Investigation 1: Use *4.3 Investigation 1 Worksheet: Drawing a Scale Diagram* on Master 4.9. Circulate to observe students' use of their compasses. Ensure that each student's compasses are screwed tightly so that the radius remains constant. Pair students with strong skills with those having difficulty.

Investigation 2: Use *4.3 Investigation 2 Worksheet: Conditions for Congruent Triangles* on Master 4.10. You may wish to have one partner construct the first triangle, while the other constructs the second. Having students work in pairs gives them the opportunity to discuss their observations about whether they are able to draw congruent triangles. Students should find that exercises 1, 3, and 5 produce only congruent triangles. For exercise 2, there are an infinite number of possible triangles. For exercise 4, there are two possible triangles.

Discussing the Ideas: **1)** There is only one triangle because all side lengths are given. **2a)** Yes. The sides will meet at the same angles in both triangles, so the shapes are the same. **b)** No. Even though the angles are equal, the sides of one triangle may be longer than the corresponding sides of the other triangle. **c)** No. Two equal angles are not a sufficient condition for congruence.

Exercises: Ensure students understand that if $\Delta ABC \cong \Delta DEF$, $\angle A$ corresponds to $\angle D$, $\angle B$ corresponds to $\angle E$, and $\angle C$ corresponds to $\angle F$. Check that students name the triangles correctly in exercises 6 and 7. For students having difficulty identifying congruent triangles, or naming corresponding equal parts, have them use tracing paper or paper cutouts to see how the first triangle matches the second.

Communicating the Ideas: Supply students with home design magazines or newspapers.

4.4 Similar Figures, page 159

Curriculum Correlation

Students will demonstrate an understanding of and apply proportions within similar triangles.

Suggested Materials

Masters 4.3, 4.11; ruler; protractor; scientific calculator

Lesson Focus

Students will apply proportions to describe and solve similar figures.

Lesson Notes

Warm-Up

- Warm-Up 4.4: Proportions on Master 4.3

Investigation: Use *4.4 Investigation Worksheet: Similar Figures* on Master 4.11. This *Investigation* helps students realize that they cannot assume congruency when figures are similar.

Example 1: Ensure students understand that since rectangles have 90º angles, we can define similarity in terms of sides only. With other figures, corresponding angles must be the same to establish similarity.

Discussing the Ideas: **1)** Measure the length and width of each rectangle. If the length-to-width ratios are equal, the rectangles are similar. **2)** They have the same shape. **3)** Yes. They have the same shape and size, so ratios of corresponding sides will be equal.

Exercises: For exercise 5b, suggest that students simplify $\frac{10.5}{4.5}$ as $\frac{105}{45}$.

Show students that there are several ways to set up a proportion to determine similarity. They can use one of the following:

- the ratio of two sides of one figure with the ratio of the corresponding sides of another figure
- the ratio of pairs of corresponding sides of two figures

For exercise 6, students can write either $\frac{AC}{4.8} = \frac{0.9}{1.2}$ or $\frac{AC}{0.9} = \frac{4.8}{1.2}$.

For exercise 7, students may be surprised that the 59% area reduction is different from the 77% reduction applied to the linear dimensions. Ask students if they can explain how 59% is obtained from 77%? (*77% × 77% = 0.77 × 0.77 = 0.5929, or about 59%*)

Extension: Have interested students investigate the *golden ratio* and the *golden rectangle*. This ratio is often called the "divine proportion" and was used by many Greek and Renaissance architects. It has been used by artists of many periods to divide their canvasses into pleasing proportions. A *golden rectangle* with length *a* and width *b* will exhibit the *golden ratio* $\frac{a}{b} = \frac{b}{a-b}$ ($\doteq 1.618 : 1$).

Communicating the Ideas: Two figures are similar when the ratios of corresponding lengths are equal. Not all rectangles are similar because the length-to-width ratio cannot be generalized. However, all squares are similar because this ratio is always 1 to 1 (1 : 1).

4.5 Conditions for Similarity, page 164

Curriculum Correlation

Students will demonstrate an understanding of and apply the properties of similar triangles.

Students will relate congruence and similarity of triangles.

Suggested Materials

Masters 4.4, 4.12, 4.13; ruler; protractor; scientific calculator

Lesson Focus

Students will identify similar triangles and find unknown measures in pairs of similar triangles.

Lesson Notes

Warm-Up

- Warm-Up 4.5: Angle Relationships on Master 4.4

For all work in this section, ensure students label triangles carefully. If ΔABC is similar to ΔMNP, this indicates that ∠A corresponds to ∠M, ∠B to ∠N, and ∠C to ∠P. The symbol ~ is used to show similarity.

Investigation 1: Use *4.5 Investigation 1 Worksheet: Identifying Similar Triangles* on Master 4.12. Start by reviewing proportions that involve three-term ratios. Consider $6 : 4 : 2 = 18 : x : y$ or $\frac{6}{18} = \frac{4}{x} = \frac{2}{y}$. Using the first two ratios, we obtain the solution $x = 12$. Using the first and third ratios, we obtain the solution $y = 6$. Remind students that for any three-term ratio, equivalent ratios can be found by multiplying or dividing each term by a constant.

Investigation 2: Use *4.5 Investigation 2 Worksheet: Properties of Similar Triangles* on Master 4.13.
Example 3: In part a, some students may have difficulty seeing the two distinct triangles. Encourage students to draw ΔDGH and ΔDEF separately. Part b may be difficult for some students. Ensure that they understand that ∠H = ∠F and ∠G = ∠E. Mark angles at D as ∠GDH and ∠EDF. To mark them as ∠D is ambiguous.

Discussing the Ideas: Determine whether the triangles have three pairs of equal angles, or whether all corresponding sides are proportional.

Exercises: Exercise 2 allows students to practise recording similarity in such a way that the correct correspondence is given. For example, students should write that ΔABC is similar to ΔEDF. Then, it is clear that AB corresponds to ED, BC corresponds to DF, and AC corresponds to EF.
For exercises 8, 9, 10, 13, and 15, encourage those students having difficulty to draw the triangles as separate triangles.
For exercise 10, some students may use the proportion 6 : 3 = 12 : ST. Since this proportion does not involve corresponding sides of similar triangles, it is not a result based on the work established in this chapter. However, it is true by what is often called the "Side-Splitting Theorem." The proof of this theorem is beyond the abilities of most grade 9 students, but you may decide to let students use it.

Communicating the Ideas: **a)** False. Similar triangles are not necessarily congruent. The triangles in *Example 1* are similar, but not congruent**. b)** True. Since congruent triangles have three pairs of equal angles, they are similar.

4.6 Solving Problems Using Similar Triangles, page 173

Curriculum Correlation

Students will demonstrate an understanding of and apply the properties of similar triangles.

Students will relate congruence and similarity of triangles.

Suggested Materials

Master 4.4; ruler; scientific calculator

Lesson Focus

Students will use properties of similar triangles to find unknown distances in applied situations.

Lesson Notes

Warm-Up

- Warm-Up 4.6: Solving for Angles on Master 4.4

This section is important because it answers the question, "Why do we need to learn this?" Students will see that the examples and exercises present real-life situations where problems can be solved using the power of the properties of similar triangles. You may decide to spend several days on this section, since visualizing similar triangles can be challenging for many students.

Example 1: An overhead transparency that shows only the two triangles may clarify this example for students.

Example 2: Ask students why a golfer would want to know the distance across the pond. (*Most golfers know how far they can drive a ball, and they would not want the ball to go into the water*.)

Discussing the Ideas: **1)** The angles in any triangle add to 180º. If we know the measures of two angles, we can find the third. **2a)** Side lengths of similar triangles are proportional. **b)** Corresponding angles of similar triangles are equal. **3a)** If we know that AB is parallel to CD, then $\angle A = \angle D$ and $\angle C = \angle B$. **b)** If we know that PQ is parallel to RS, then $\angle TPQ = \angle TRS$ and $\angle TQP = \angle TSR$.

Exercises: For exercise 2, parts b and c, ensure students identify the correct similar triangles.

For exercise 8, if students have difficulty constructing the diagram, refer them to exercise 5. In both exercises, students should assume that the sun's rays are parallel. A common student mistake is to forget to put the shadow on the ground, but instead to think that the shadow is the hypoteneuse of the right triangle.

For exercise 9, ensure students convert all measures to a common metric measure. You may want to review the relationship between millimetres, centimetres, and metres.

For exercise 16, assume that each row includes all three sections.

Communicating the Ideas: Exercises 4 to 16 give a variety of situations where similar triangles can be used to calculate distances that cannot be measured.

4.7 Mathematical Modelling: Solar Eclipses, page 179

Curriculum Correlation

Students will demonstrate an understanding of and apply the properties of similar triangles.

Suggested Materials

Master 4.5; paper; measuring tape or metre sticks; compasses; tape; scissors; scientific calculator

Lesson Focus

Students will use similar triangles to model a total eclipse of the sun.

Lesson Notes

Warm-Up

- Warm-Up 4.7: Using Ratios on Master 4.5

The eclipse experiment models a total solar eclipse. The sizes of the circles are not critical as long as one circle is much larger than the other. Since there is a huge difference between the size of the moon and the size of the sun, the eclipse experiment does not exactly model an eclipse. A realistic model would have the moon with radius 1 cm and the sun with radius 4 m. The distance between the circles would be about 860 m, and the distance from the observer to the "eclipser" would be about 2 m. Telling students these numbers will help them understand the relative distance from Earth to the moon and Earth to the sun.

Exercises: For exercise 2, mark the viewing position by placing an object on the floor or using masking tape.

For exercises 3 and 4, the eclipser should hold his or her hand steady once the correct position is found. The observer should locate a point on the floor directly below the small circle and mark the floor with an object or tape.

In exercise 7, students should discover that the ratio is constant. The constant is equal to the ratio of the radii of the two circles. Have students use similar triangles to explain why this is so.

For exercises 8 and 9, if e = eclipsing distance and v = viewing distance, students should develop a relationship such as $\frac{e}{v} = \frac{\text{radius of small circle}}{\text{radius of large circle}}$, or $e = \frac{\text{radius of small circle}}{\text{radius of large circle}} \times v$. This can be written as the linear function $e = kv$, where k equals the ratio of the radii of the two circles. Have students use the equation to predict the eclipsing distance for different viewing distances.

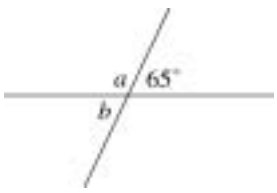
For exercise 10, students will replace e with the distance to the moon, v with the distance to the sun, the radius of the small circle with the radius of the moon, and the radius of the large circle with the radius of the sun. Explain to students that although the radii of the moon and sun are constant, the distances to the moon and sun may vary because their orbits are ellipses, not circles.

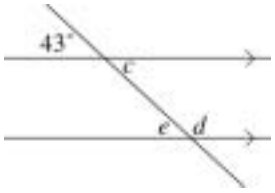
Communicating the Ideas: Circular models of the moon and sun can be used to model a solar eclipse. Cut out a large circle to represent the sun and tape it to a wall. Cut out a much smaller circle for the moon. You stand about 3 m from the wall. Have a helper stand between you and the wall, holding the small circle between your eyes and the large circle. The helper places the small circle so that it blocks your view of the large circle completely.

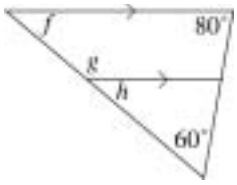
Chapter 4 Necessary Skills

Parallel Lines, Triangles, and Related Angles

1. Determine the angle measure indicated by each letter.







2. In exercise 1, identify the name given to each pair of angles.

a) c and d

b) f and h

c) c and e

3. a) Two angles whose sum is 180º are _____ angles.

b) Two angles whose sum is 90º are _____ angles.

Ratio and Proportion

1. Solve for x and y.

a) $\frac{x}{3} = \frac{4}{12}$ **b)** $\frac{3}{7} = \frac{x}{28}$ **c)** $\frac{20}{x} = \frac{3}{6} = \frac{5}{y}$ **d)** $\frac{5}{x} = \frac{x}{20}$

2. Solve for x and y. Round answers to one decimal place.

a) $\frac{4}{x} = \frac{5}{6}$ **b)** $\frac{3}{5} = \frac{2}{y}$ **c)** $\frac{12}{7} = \frac{3}{x} = \frac{2}{y}$ **d)** $\frac{x}{9} = \frac{5}{11} = \frac{y}{4}$

3. The ratio of girls to boys in a class is 5 : 7. If there are 21 boys in the class, how many girls are there?

Warm-Up 4.1

Equations and Percent

1. Solve for x.

a) $x = 2(10)$ **b)** $12 = 3x$ **c)** $8(2) = 4x$

d) $x = \frac{10}{2}$ **e)** $\frac{x}{3} = 5$ **f)** $12 = \frac{x}{6}$

2. Calculate each value.

a) 80% of 120 **b)** 35% of 90 **c)** 40% of 220

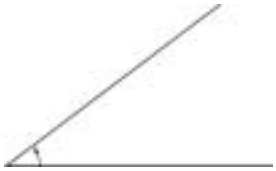
d) 10% of 78.3 **e)** 15% of 12.3 **f)** 20% of 95

Warm-Up 4.2

Angles

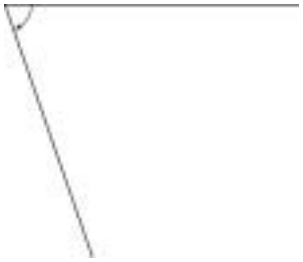
1. Measure each angle.

a)



b)

c)



d)

2. Construct an angle with each measure.

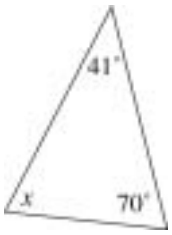
a) 40° **b)** 100° **c)** 150° **d)** 35°

Warm-Up 4.3

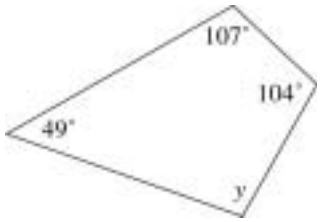
Determining Angle Measures

1. Determine the measure of each indicated angle.

a)



b)



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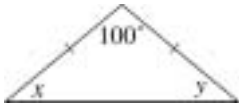
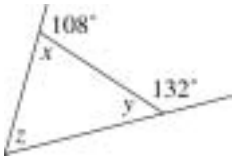
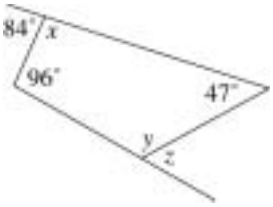


2. Determine the values of x, y, and z.

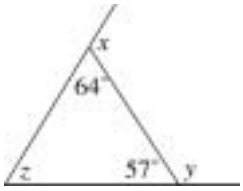
a)



b)



c)



Warm-Up 4.4

Proportions

1. Solve the following proportions for x.

a) $\frac{2}{3} = \frac{x}{21}$ **b)** $\frac{4}{x} = \frac{48}{36}$ **c)** $\frac{2}{5} = \frac{x}{30}$

d) $\frac{7}{x} = \frac{21}{15}$ **e)** $\frac{x}{96} = \frac{5}{12}$ **f)** $\frac{5}{6} = \frac{35}{x}$

2. Solve the following proportions for x. Round answers to one decimal place.

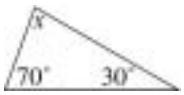
a) $\frac{2}{5} = \frac{x}{12}$ **b)** $\frac{x}{5} = \frac{10}{3}$ **c)** $\frac{7}{9} = \frac{x}{4}$

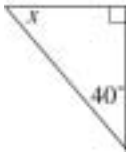
d) $\frac{x}{5.2} = \frac{5}{6}$ **e)** $\frac{2.5}{x} = \frac{7.1}{9}$ **f)** $\frac{6.3}{9.2} = \frac{3.5}{x}$

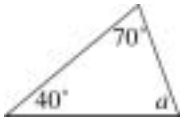
Warm-Up 4.5

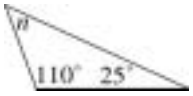
Angle Relationships

1. Solve for the unknown in each triangle.

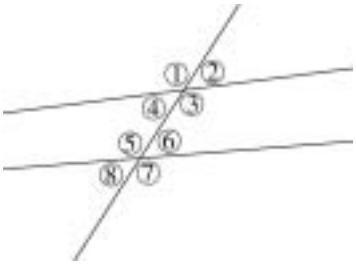
a) 

b) 

c) 

d) 

2. Use the diagram below.



a) Name pairs of interior angles.

b) Name pairs of corresponding angles.

c) Name pairs of alternate angles.

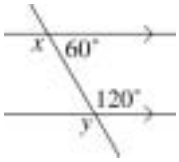
d) Name pairs of angles that should add to 180°. Explain how you know.

e) Name pairs of angles that should be equal.

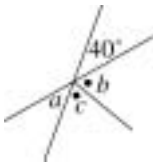
Warm-Up 4.6

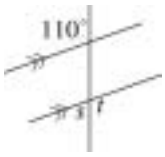
Solving for Angles

1. Solve for the unknowns in each diagram.

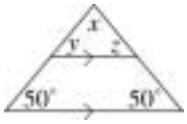
a) 

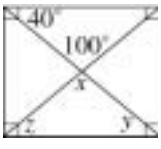
b) 

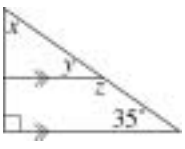
c) 

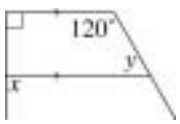
d) 

2. Solve for the unknowns in each diagram.

a) 

b) 

c) 

d) 

Warm-Up 4.7

Using Ratios

1. Write each ratio in simplest form.

a) 40 : 12 **b)** 5 : 65 **c)** 32 : 52

d) 50 : 250 **e)** 65 : 13 **f)** 144 : 9

2. Use equivalent ratios to find the missing term in each proportion.

a) 3 : 8 = Ä: 24 **b)** Ä : 12 = 15 : 36 **c)** 9 : 8 = 144 : Ä

d) 11 : 16 = Ä : 32 **e)** 5 : 7 = 5.5 : Ä **f)** Ä : 4 = 21 : 12

3. For every 35 swimmers at a pool, there should be one lifeguard. Determine the number of lifeguards needed for each number of swimmers.

a) 70 **b)** 140 **c)** 245

4.1 Investigation 3 Worksheet: Constructing Triangles

Name: _____ Date: _____

1. Construct each triangle.

a) Side 6 cm between angles 36° and 46°

b) Angle 96° between sides 7.5 cm and 6.5 cm

c) Side 7.4 cm between angles 53° and 53°

d) Angle 114° between sides 5.7 cm and 4.6 cm

e) Side 8.3 cm between angles 124° and 14°

f) Angle 60° between sides 7.9 cm and 7.9 cm

g) Side 9.2 cm between angles 72° and 82°

h) Angle 75° between sides 5.4 cm and 5.4 cm

i) Side 10.4 cm between angles 60° and 60°

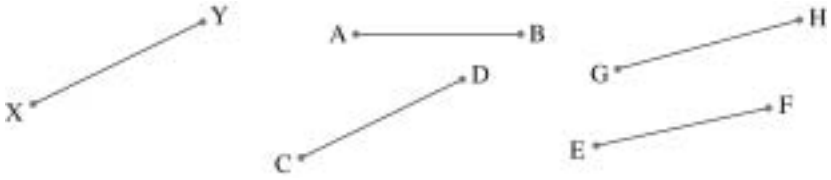
j) Angle 108° between sides 6.8 cm and 9.7 cm

2. Compare each triangle you constructed in exercise 1 with those of other students. For each set of measurements, are all the triangles the same? If they are not the same, explain how they are different.

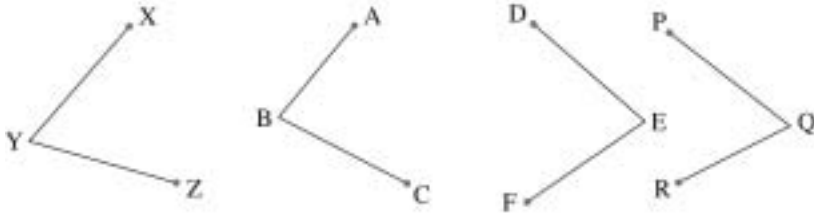
4.2 Investigation 2 Worksheet: Congruent Line Segments and Angles

Name: _____ **Date:** _____

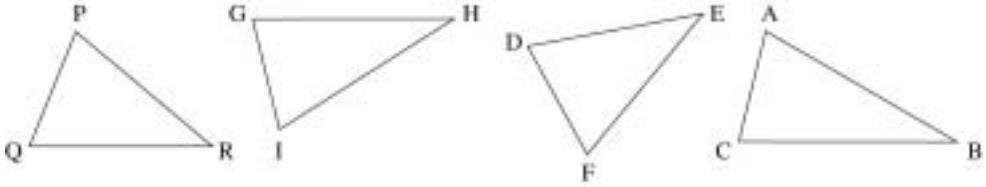
1. Use tracing paper to determine which line segment is congruent to XY.



2. Use tracing paper to determine which angle is congruent to ∠XYZ.



3. Use tracing paper to determine which triangle is congruent to ΔPQR.



4. Check your answers to exercises 1 to 3 with other students. If you do not agree on the answers, try to find the mistakes.

5. a) In exercise 1, how could you find which line segment is congruent to XY without tracing?

b) How are congruent line segments *equal?*

6. a) In exercise 2, how could you find which angle is congruent to $\angle XYZ$ without tracing?

b) How are congruent angles *equal?*

7. a) In exercise 3, how could you find which triangle is congruent to ΔPQR without tracing?

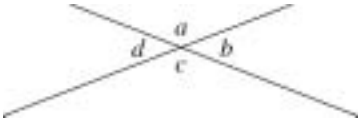
b) If you know that two triangles are congruent, what can you say about their sides and their angles?

4.2 Investigation 3 Worksheet: Opposite Angles

Name: _____ Date: _____

Work in a group. You will need a protractor.

1. Draw two lines that intersect. Label the angles as shown.



2. Measure the angles. Record the answers. What do you notice?

3. Repeat exercises 1 and 2 for a different pair of interesting lines.

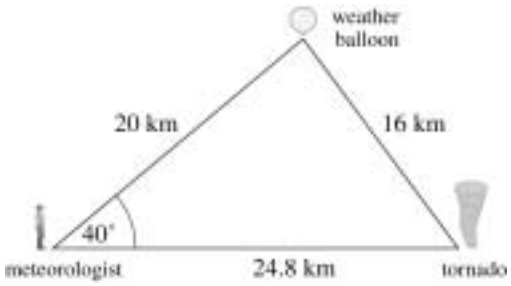
4. Compare your results with those of the other students in your group.

5. Did you find pairs of congruent angles?

6. Write a statement about the measures of the angles formed by two intersecting lines.

4.3 Investigation 1 Worksheet: Drawing a Scale Diagram

Name: _____ Date: _____



STEP 1

Use a scale of 1 cm to represent 2 km. How long should you draw the line segment:

a) joining the meteorologist to the balloon?

b) joining the balloon to the tornado?

STEP 2

Use a line segment to represent the line between the meteorologist, M, and the tornado. Use a protractor to construct an angle of 40° at M.

STEP 3

Measure 10 cm along the line from M to the balloon B. Set your compasses to 8 cm. Place the compasses point on B and draw a circle.

STEP 4

Join B to each point where the circle intersects the line through M and the tornado. Label these points X and Y.

1. Measure MX and MY. What are the two possible distances that the tornado was from the meteorologist?

2. Which triangle should the meteorologist have drawn to model the position of the tornado?

3. How far away was the tornado when the meteorologist received the information from the balloon?

4. What further information did the meteorologist need before he drew his scale diagram?

4.3 Investigation 2 Worksheet: Conditions for Congruent Triangles

Name: _____ Date: _____

Work with a partner. You will need compasses, a protractor, and a ruler. For each exercise, you are given the same measurements for two triangles. Use these measurements to try to construct two triangles that are not congruent.

1. AB = 6 cm, BC = 8 cm, AC = 12 cm DE = 6 cm, EF = 8 cm, DF = 12 cm

Can you draw ΔABC so that it is not congruent to ΔDEF?

2. $\angle G = 65°$, $\angle H = 85°$, $\angle I = 30°$ $\angle J = 65°$, $\angle K = 85°$, $\angle M = 30°$

Can you draw ΔGHI so that it is not congruent to ΔJKM?

3. NP = 8 cm, PQ = 5 cm, $\angle P = 50°$ RS = 8 cm, ST = 5 cm, $\angle S = 50°$

Can you draw ΔNPQ so that it is not congruent to ΔRST?

4. UV = 7 cm, UW = 4 cm, $\angle V = 30°$ XY = 7 cm, XZ = 4 cm, $\angle Y = 30°$

Can you draw ΔUVW so that it is not congruent to ΔXYZ?

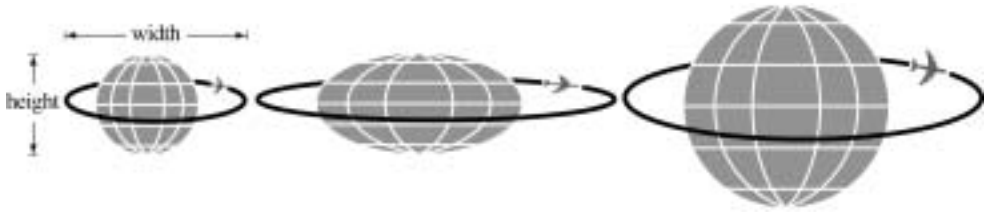
5. $\angle BAC = 62°$, $\angle ABC = 80°$, $BA = 6$ cm $\angle EDF = 62°$, $\angle DEF = 80°$, $ED = 6$ cm

Can you draw ΔABC so that it is not congruent to ΔDEF?

6. Compare your results with those of other students. Which measurements of a triangle would you have to be given so that only one triangle can be drawn?

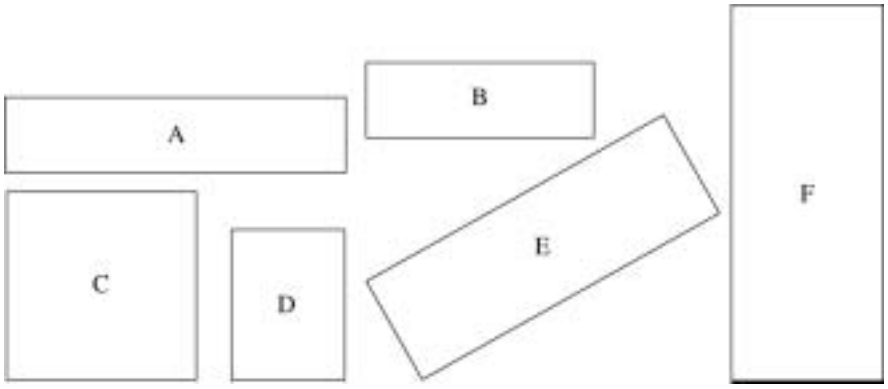
4.4 Investigation Worksheet: Similar Figures

Name: _____ **Date:** _____



1. For each logo, measure the height and the width. Calculate its height-to-width ratio to one decimal place. What do you discover?

2. a) Repeat these measurements with the rectangles below.



b) Complete this table.

Rectangle	Length	Width	$\frac{\text{Length}}{\text{Width}}$
A			
B			
C			
D			
E			
F			

3. Calculate and record the length : width ratio for each rectangle. Write the ratio as a decimal.

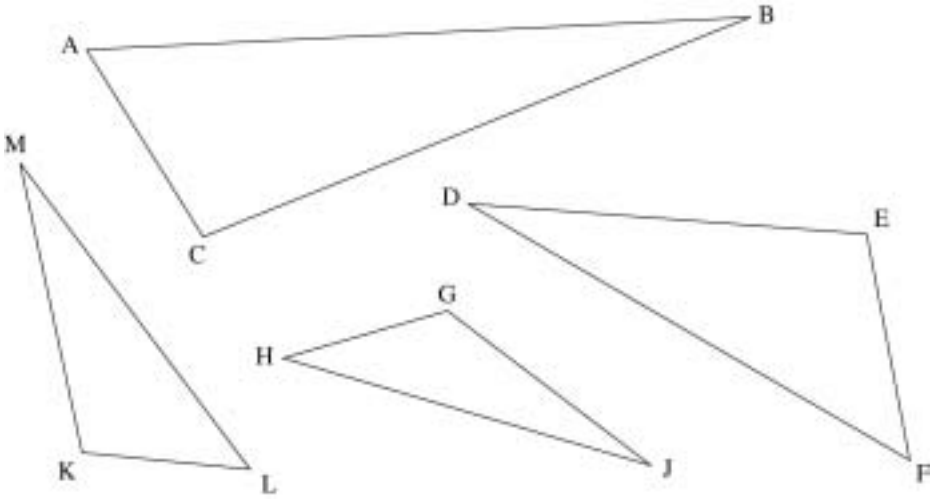
4. Which two rectangles have the same length : width ratio?

5. We say that two rectangles that have the same length : width ratio are *similar*. Similar rectangles have the same shape but not necessarily the same size. Which rectangles are similar? Explain.

4.5 Investigation 1 Worksheet: Identifying Similar Triangles

Name: _____ Date: _____

1. Measure the sides of each triangle. Write each length to the nearest 0.5 mm.



2. Arrange the lengths of the sides of each triangle in order from longest to shortest.

3. Write the lengths of the sides of each triangle as a three-term ratio.

4. For each triangle, divide each term of the ratio by the last term of the ratio. That is, write an equivalent ratio with last term 1. Where necessary, round the quotient to 2 decimal places.

5. Identify the two triangles that have the same three-term ratio.

6. Identify the two triangles that are similar.

7. Compare your results with those of other students. Did all of you find the same two similar triangles?

4.5 Investigation 2 Worksheet: Properties of Similar Triangles

Name: _____ Date: _____

1. Draw any ΔABC. Measure the angles.

2. Measure the lengths of the sides of ΔABC.

3. Multiply each length in step 2 by 1.5.

4. Construct ΔEFG with side lengths equal to those in step 3.

5. Measure the angles of ΔEFG.

6. Record your results in these tables.

Triangle ABC

Side	Length	Angle	Measure
AB		A	
BC		B	
CA		C	

Triangle EFG

Side	Length	Angle	Measure
EF		E	
FG		F	
GE		G	

7. Are the two triangles you drew similar? How do you know?

8. What do you notice about the angle measures in the two triangles?

9. Compare your results with those of other students. Did all of you get the same answers to the questions in steps 7 and 8?

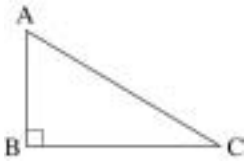
Chapter 4 Test

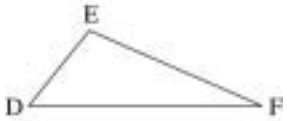
Name: _____ **Date:** _____

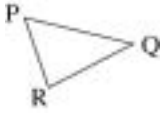
Provide complete solutions to achieve full marks. If you encounter any difficulties with an exercise, leave it and return to it after you have attempted the remainder of the test.

1. Find an example of each type of triangle.

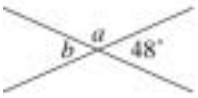
a) isosceles **b)** right **c)** obtuse **d)** scalene

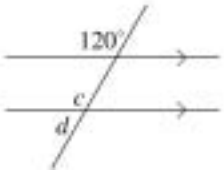


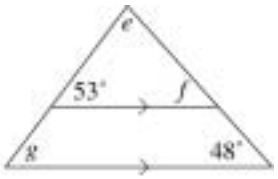




2. Determine the angle measure indicated by each letter.

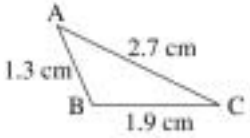


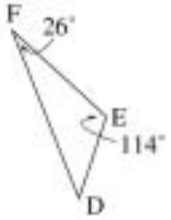




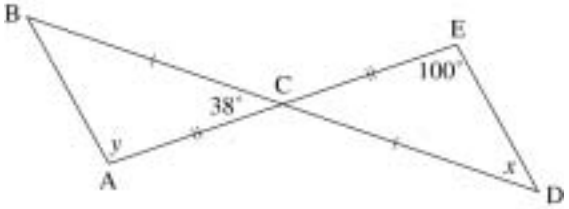
3. In the diagram below, ΔABC is congruent to ΔDEF. Determine each measure.

a) DF **b)** $\angle FDE$ **c)** $\angle ACB$





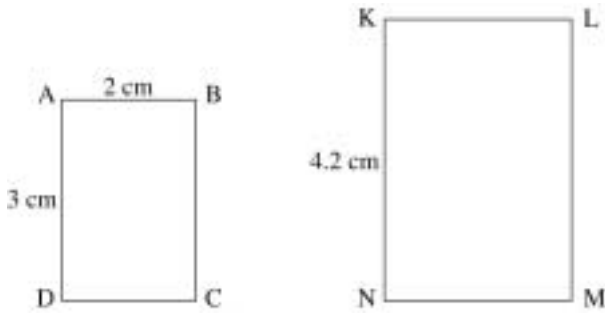
4. a) In the figure below, name a pair of congruent triangles.



b) Give a reason why the triangles are congruent.

c) Solve for x and y.

5. The two rectangles in this diagram are similar.



a) State the ratio of the corresponding sides of the two rectangles.

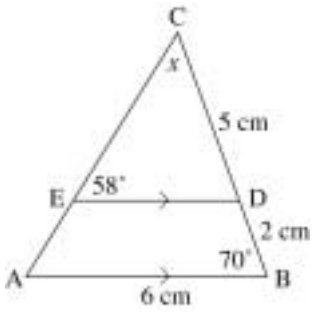
b) Determine the length of KL.

c) Determine the ratio of the areas of the two rectangles.

6. a) State three conditions that would result in two triangles being congruent.

b) State two conditions that would result in two triangles being similar.

7. a) Name a pair of similar triangles.



b) Determine the value of x.

c) Determine the length of ED. Round answer to 1 decimal place.

8. A pole is 2.8 m high. It casts a shadow that is 1.2 m long. At the same time, a nearby building casts a shadow that is 7.6 m long. How high is the building? Include a labelled diagram with your solution. Round answer to 1 decimal place.

9. Two isosceles triangles are always similar.
Do you agree with the above statement? Give reasons and an example to support your answer.

Answers to Masters

Master 4.1

Parallel Lines, Triangles, and Related Angles

1. a) $a = 115^\circ$, $b = 65^\circ$

b) $c = 43^\circ$, $d = 137^\circ$, $e = 43^\circ$

c) $f = 40^\circ$, $g = 140^\circ$, $h = 40^\circ$

2. a) Interior **b)** Corresponding **c)** Alternate

3. a) Supplementary **b)** Complementary

Ratio and Proportion

1. a) $x = 1$ **b)** $x = 12$ **c)** $x = 40$, $y = 10$ **d)** $x = \pm 10$

2. a) $x = 4.8$ **b)** $y = 3.3$ **c)** $x = 1.8$, $y = 1.2$ **d)** $x = 4.1$, $y = 1.8$

3. 15

Master 4.2

Warm-Up 4.1

1. a) 20 **b)** 4 **c)** 4 **d)** 5

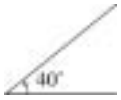
e) 15 **f)** 72

2. a) 96 **b)** 31.5 **c)** 88 **d)** 7.83

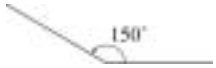
e) 1.845 **f)** 19

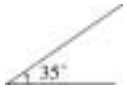
Warm-Up 4.2

1. a) 37° **b)** 102° **c)** 71° **d)** 155°

2. a) 

b) 

c) 

d) 

Master 4.3

Warm-Up 4.3

1. a) $x = 69^\circ$ **b)** $y = 100^\circ$ **c)** $x = y = 40^\circ$

2. a) $x = 72^\circ$, $y = 48^\circ$, $z = 60^\circ$ **b)** $x = 96^\circ$, $y = 121^\circ$, $z = 59^\circ$ **c)** $x = 116^\circ$, $y = 123^\circ$, $z = 59^\circ$

Warm-Up 4.4

1. a) 14 **b)** 3 **c)** 12

d) 5 **e)** 40 **f)** 42

2. a) 4.8 **b)** 16.7 **c)** 3.1

d) 4.3 **e)** 3.2 **f)** 5.1

Master 4.4

Warm-Up 4.5

1. a) 80º **b)** 50º **c)** 70º **d)** 45º

2. a) 4 and 5, 3 and 6 **b)** 1 and 5, 4 and 8, 2 and 6, 3 and 7

c) 3 and 5, 4 and 6 **d)** 1 and 2, 3 and 4, 5 and 6, 7 and 8, 1 and 4, 2 and 3, 5 and 8, 6 and 7
They add to 180º because they are supplementary.

e) 1 and 3, 2 and 4, 5 and 7, 6 and 8, 1 and 5, 4 and 8, 2 and 6, 3 and 7, 1 and 7, 2 and 8

Warm-Up 4.6

1. a) $x = 120^\circ$, $y = 120^\circ$ **b)** $k = 125^\circ$, $m = 55^\circ$, $n = 55^\circ$

c) $a = 40^\circ$, $b = c = 70^\circ$ **d)** $s = 70^\circ$, $t = 110^\circ$

2. a) $x = 80^\circ$, $y = z = 50^\circ$ **b)** $x = 100^\circ$, $y = 40^\circ$, $z = 40^\circ$

c) $x = 55^\circ$, $y = 35^\circ$, $z = 145^\circ$ **d)** $x = 90^\circ$, $y = 60^\circ$

Master 4.5

Warm-Up 4.7

1. a) 10 : 3 **b)** 1 : 13 **c)** 8 : 13
d) 1 : 5 **e)** 5 : 1 **f)** 16 : 1

2. a) 9 **b)** 5 **c)** 128
d) 22 **e)** 7.7 **f)** 7

3. a) 2 **b)** 4 **c)** 7

Master 4.14

Chapter 4 Test

1. a) ΔLMN **b)** ΔABC **c)** ΔDEF **d)** ΔPQR, or ΔABC, or ΔDEF

2. $a = 132^\circ$, $b = 48^\circ$, $c = 120^\circ$, $d = 60^\circ$, $e = 79^\circ$, $f = 48^\circ$, $g = 53^\circ$

3. a) 2.7 cm **b)** 40° **c)** 26°

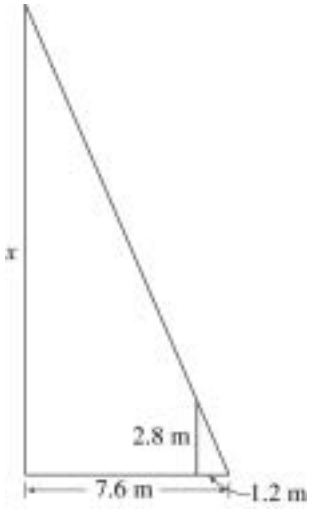
4. a) ΔABC is congruent to ΔEDC **b)** SAS **c)** $x = 42^\circ$, $y = 100^\circ$

5. a) 1.4 : 1 or 1 : 1.4 **b)** 2.8 cm **c)** 1.96 : 1 or 1 : 1.96

6. a) SSS, SAS, ASA
b) The ratios of corresponding sides are equal, or corresponding angles are equal.

7. a) ΔCAB is similar to ΔCED. **b)** 52° **c)** 4.3 cm

8. 17.7 m



9. Not true. The two equal angles in an isosceles triangle can have many different values. For example, in ΔABC, ∠A = ∠B could be 50°, while in ΔDEF, ∠D = ∠E could be 65°.