Appendix

Algebra of the *ISLM* Model

The use of algebra to analyze the *ISLM* model allows us to extend the multiplier analysis in Chapter 22 and to obtain many of the results of Chapters 22 and 23 very quickly.

Basic Closed-Economy *ISLM* Model

The goods market can be described by the following equations:

Consumption function:	$C = \overline{C} + mpc\,(Y - T)$	(1)
Investment function:	$I = \overline{I} - di$	(2)
Taxes:	$T = \overline{T}$	(3)
Government spending:	$G = \overline{G}$	(4)
Goods market equilibrium condition:	$Y = Y^{ad} = C + I + G$	(5)

The money market is described by these equations:

Money demand function:	$M^d = \overline{M}^d + eY - fi$	(6)
Money supply:	$M^s = \overline{M}$	(7)
Money market equilibrium condition:	$M^d = M^s$	(8)

The uppercase terms are the variables of the model; $\overline{G}$, $\overline{T}$, and $\overline{M}$ are the values of the policy variables that are set exogenously (outside the model); and $\overline{C}$, $\overline{I}$, and $\overline{M}^d$ are autonomous components of consumer expenditure, investment spending, and money demand that are also determined exogenously (outside the model). Except for the interest rate i, the lowercase terms are the parameters, the givens of the model, and all are assumed to be positive. The definitions of these variables and parameters are as follows:

C = consumer spending
I = investment spending
$G = \overline{G}$ = government spending
Y = output
$T = \overline{T}$ = taxes

M^d = money demand
$M^s = \overline{M}$ = money supply
i = interest rate
$\overline{C}$ = autonomous consumer spending
d = interest sensitivity of investment spending
$\overline{I}$ = autonomous investment spending related to business confidence
$\overline{M}^d$ = autonomous money demand
e = income sensitivity of money demand
f = interest sensitivity of money demand
mpc = marginal propensity to consume

IS and *LM* Curves

Substituting for *C, I,* and *G* in the goods market equilibrium condition and then solving for *Y,* we obtain the *IS* curve:

$$Y = \frac{1}{1-mpc}(\overline{C} + \overline{I} - mpc\overline{T} + \overline{G} - di) \qquad (9)$$

Solving for *i* from Equations 6, 7, and 8, we obtain the *LM* curve:

$$i = \frac{\overline{M}^d - \overline{M} + eY}{f} \qquad (10)$$

Solution of the Model

The solution to the model occurs at the intersection of the *IS* and *LM* curves, which involves solving for *Y* and *i* simultaneously, using Equations 9 and 10, as follows:

$$Y = \frac{1}{1-mpc+de/f}\left(\overline{C} + \overline{I} - mpc\overline{T} + \overline{G} - \frac{d\overline{M}^d}{f} + \frac{d\overline{M}}{f}\right) \qquad (11)$$

$$i = \frac{1}{f(1-mpc)+d}\left[e(\overline{C} + \overline{I} - mpc\overline{T} + \overline{G}) + \overline{M}^d(1-mpc) - \overline{M}(1-mpc)\right] \qquad (12)$$

Implications

The conclusions reached with these algebraic solutions are the same as those reached in Chapters 22 and 23; for example:

1. Because all the coefficients are positive, Equation 11 indicates that a rise in $\overline{C}$, $\overline{I}$, $\overline{G}$, and $\overline{M}$ leads to a rise in Y and that a rise in $\overline{T}$ or $\overline{M}^d$ leads to a fall in Y.

2. Equation 12 indicates that a rise in $\overline{C}$, $\overline{I}$, $\overline{G}$, and $\overline{M}^d$ leads to a rise in i and that a rise in $\overline{M}$ or $\overline{T}$ leads to a fall in i.

3. As f, the interest sensitivity of money demand, increases, the multiplier term

$$\frac{1}{1 - mpc + de/f}$$

increases, and so fiscal policy ($\overline{G}$, $\overline{T}$) has more effect on output; conversely, the term multiplying $\overline{M}$,

$$\frac{d}{f}\left(\frac{1}{1 - mpc + de/f}\right) = \frac{d}{f(1 - mpc + de)}$$

declines, so monetary policy has less effect on output.

4. By similar reasoning, as d, the interest sensitivity of investment spending, increases, monetary policy has more effect on output and fiscal policy has less effect on output.

Open-Economy *ISLM* Model

To make the basic *ISLM* model into an open-economy model, we need to include net exports in the goods market equilibrium condition so that Equation 5 becomes Equation 5':

$$Y = Y^{ad} = C + I + G + NX \qquad (5')$$

As the discussion in Chapter 23 suggests, the net exports and exchange rate relations can be written

$$NX = \overline{NX} - hE \qquad (13)$$

$$E = \overline{E} + ji \qquad (14)$$

where

NX = net exports

$\overline{NX}$ = autonomous net exports

h = exchange rate sensitivity of net exports

E = exchange rate (value of domestic currency)

$\overline{\overline{E}}$ = autonomous exchange rate

j = interest sensitivity of exchange rate

Substituting for net exports in the goods market equilibrium condition (Equation 5') using the net exports and exchange rate relations and then solving for Y as in the basic model, we obtain the open-economy *IS* curve:

$$Y = \frac{1}{1-mpc}\left[\overline{C}+\overline{I}-mpc\overline{T}+\overline{G}+\overline{NX}-h\overline{\overline{E}}-(d+hj)i\right] \qquad (15)$$

The *LM* curve is the same as in the basic model, and so the solutions for Y and i are as follows:

$$Y = \frac{1}{1-mpc+(d+hj)e/f}\times\left(\overline{C}+\overline{I}-mpc\overline{T}+\overline{G}-\frac{d+hj}{f}\overline{M}^{d}+\frac{d+hj}{f}\overline{M}+\overline{NX}-h\overline{\overline{E}}\right)$$

$$(16)$$

$$i = \frac{1}{f(1-mpc)+(d+hj)e}\times\left[e\left(\overline{C}+\overline{I}-mpc\overline{T}+\overline{G}+\overline{NX}-h\overline{\overline{E}}\right)+\overline{M}^{d}(1-mpc)-\overline{M}(1-mpc)\right]$$

$$(17)$$

Implications

1. As the *IS* curve in Equation 15 indicates, including net exports in aggregate demand provides an additional reason for the negative relationship between Y and i (the downward slope of the *IS* curve). This additional reason for the negative relationship of Y and i is represented by hj in the term $-(d+hj)i$.

2. Equations 16 and 17 indicate that all the results we found for the basic model still hold.

3. Equation 16 indicates that a rise in $\overline{NX}$ leads to a rise in Y and that an autonomous rise in the value of the domestic currency $\overline{\overline{E}}$ leads to a decline in Y.

4. Equation 17 indicates that a rise in $\overline{NX}$ leads to a rise in i and that a rise in $\overline{E}$ leads to a decline in i.