appendix 1 to chapter 22

A Mathematical Treatment of the Baumol-Tobin and Tobin Mean-Variance Models

The derivation of a money multiplier for the M2 definition of money requires only slight modifications to the analysis in the chapter. The definition of M2 is:

$$M2 = C + D + T + MMF$$

where C = currency in circulation
D = checkable deposits
T = time and savings deposits
MMF = money market mutual funds

We again assume that all desired quantities of these variables rise proportionally with checkable deposits so that the equilibrium ratios

c = currency ratio, C/D
t = time deposit ratio, T/D
f = money market fund ratio, MMF/D

set by depositors and the desired reserve ratio r set by banks are treated as constants. Replacing C by $c \times D$, T by $t \times D$, and MMF by $f \times D$ in the definition of M2+ just given, we get

$$\begin{aligned} M2+ &= (c \times D) + D + (t \times D)\ (f \times D) \\ &= (1 + c + t + f) \times D \end{aligned}$$

Substituting in the expression for D from Equation 2 in the chapter,[1] we have

$$M2 = \frac{1 + c + t + mm}{r + e + c} \times MB$$

To see what this formula implies about the M2+ money multiplier, we continue with the same numerical example in the chapter, with the additional information that T = \$320 billion and MMF = \$80 billion so that $t = 2$ and $mm = 0.5$. The resulting value of the multiplier for M2+ is:

$$m = \frac{1 + 0.25 + 0.5}{0.25 + 0.05} = \frac{3.75}{0.3} = 12.5$$

An important feature of the M2+ multiplier is that it is substantially above the M1+ multiplier of 4.2 that we found in the chapter. The crucial concept in understanding this difference is that a lower required reserve ratio for time deposits or money market mutual fund means that they undergo more multiple expansion because fewer reserves are needed to support the same amount of them. Time deposits and *MMF*s have a lower required reserve ratio than checkable deposits—zero—and they will therefore have more multiple expansion than checkable deposits will. Thus the overall multiple expansion for the sum of these deposits will be greater than for checkable deposits alone, and so the M2+ money multiplier will be greater than the M1+ money multiplier.

Factors That Determine the M2 Money Multiplier

Changes in *c* and *r*

The economic reasoning analyzing the effect of changes in the desired reserve ratio and the currency ratio on the M2+ money multiplier is identical to that used for the M1+ multiplier in the chapter. An increase in the desired reserve ratio r will decrease the amount of multiple deposit expansion, thus lowering the M2+ money multiplier. An increase in c means that depositors have shifted out of checkable deposits into currency, and since currency has no multiple deposit expansion, the overall level of multiple deposit expansion for M2+ must also fall, lowering the M2+ multiplier.

We thus have the same results we found for the M1 multiplier: ***The M2+ money multiplier and M2+ money supply are negatively related to the required reserve ratio r, the currency ratio c, and the excess reserves ratio e.***

Response to Changes in *t* and *f*

An increase in either t or f leads to an increase in the M2+ multiplier, because the desired reserve ratios on time deposits and money market mutual fund shares are zero and hence are lower than the desired reserve ratio on checkable deposits.

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[1] To minimize costs, the second derivative must be greater than zero. We find that it is, because:

$$\frac{d^2COSTS}{dC^2} = \frac{-2}{C^3}(-bT_0) = \frac{2bT_0}{C^3} > 0$$

[2] An alternative way to get Equation 1 is to have the individual maximize profits, which equal the interest on bonds minus the brokerage costs. The average holding of bonds over a period is just:

$$\frac{T_0}{2} - \frac{C}{2}$$

Thus profits are:

$$PROFITS = -\frac{i}{2}(T_0 - C) - \frac{bT_0}{C}$$

Then:

$$\frac{d\,PROFITS}{dC} = \frac{-i}{2} + \frac{bT_0}{C^2} = 0$$

This equation yields the same square root rule as Equation 1.

Both time deposits and money market mutual funds undergo more multiple expansion than checkable deposits. Thus a shift out of checkable deposits into time deposits or money market mutual funds, increasing t or f, implies that the overall level of multiple expansion will increase, raising the M2+ money multiplier.

A decline in t or f will result in less overall multiple expansion, and the M2+ money multiplier will decrease, leading to the following conclusion: ***The M2+ money multiplier and M2+ money supply are positively related to both the time deposit ratio t and the money market fund ratio f.***

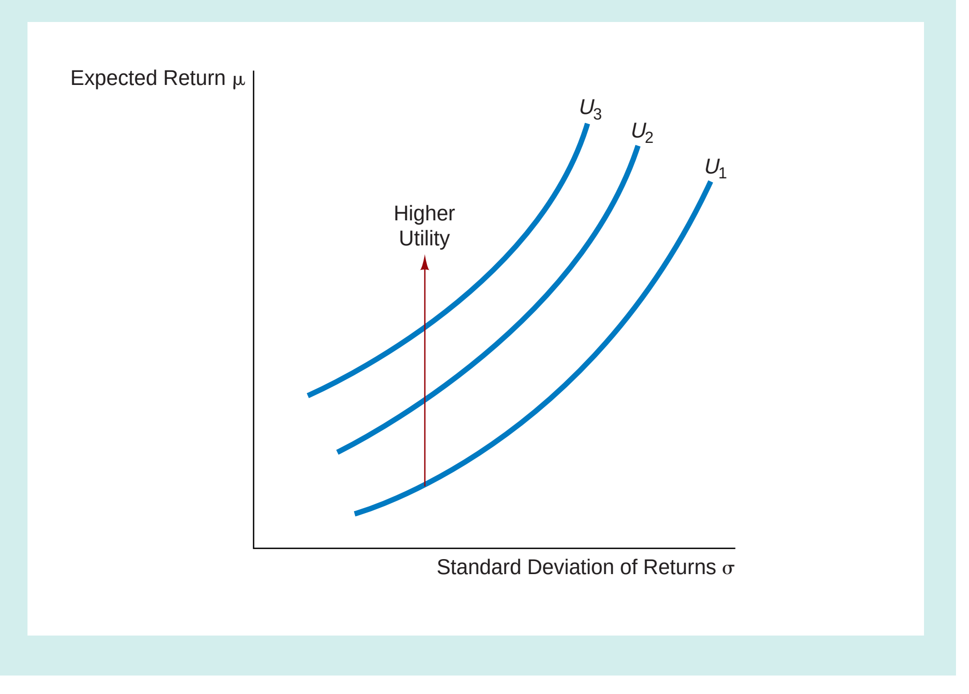
The response of the M2+ money supply to all the depositor and required reserve ratios is summarized in Table 16A-1.

Baumol-Tobin Model of Transactions Demand for Money

The basic idea behind the Baumol-Tobin model was laid out in the chapter. Here we explore the mathematics that underlie the model. The assumptions of the model are as follows:

1. An individual receives income of T_0 at the beginning of every period.
2. An individual spends this income at a constant rate, so at the end of the period, all income T_0 has been spent.
3. There are only two assets—cash and bonds. Cash earns a nominal return of zero, and bonds earn an interest rate i.

FIGURE 1 Indifference Curves in a Mean-Variace Model
The indifference curves are upward-sloping, and higher indifference curves indicate that utility is higher. In other words, $U_3 > U_2 > U_1$.



[3] This assumption is not critical to the results. If $E(g) \neq 0$, it can be added to the interest term i, and the analysis proceeds as indicated.

4. Every time an individual buys or sells bonds to raise cash, a fixed brokerage fee of b is incurred.

Let us denote the amount of cash that the individual raises for each purchase or sale of bonds as C, and n = the number of times the individual conducts a transaction in bonds. As we saw in Figure 3 in the chapter, where $T_0 = 1{,}000$, $C = 500$, and $n = 2$:

$$n = \frac{T_0}{C}$$

Because the brokerage cost of each bond transaction is b, the total brokerage costs for a period are:

$$nb = \frac{bT_0}{C}$$

Not only are there brokerage costs, but there is also an opportunity cost to holding cash rather than bonds. This opportunity cost is the bond interest rate i times average cash balances held during the period, which, from the discussion in the chapter, we know is equal to $C/2$. The opportunity cost is then:

$$\frac{iC}{2}$$

Combining these two costs, we have the total costs for an individual equal to:

$$COSTS = \frac{bT_0}{C} + \frac{iC}{2}$$

The individual wants to minimize costs by choosing the appropriate level of C. This is accomplished by taking the derivative of costs with respect to C and setting it to zero.[1] That is:

$$\frac{d\ COSTS}{dC} = \frac{-\ bT_0}{C^2} + \frac{i}{2} = 0$$

Solving for C yields the optimal level of C:

$$C = \sqrt{\frac{2bT_0}{i}}$$

Because money demand M^d is the average desired holding of cash balances $C/2$,

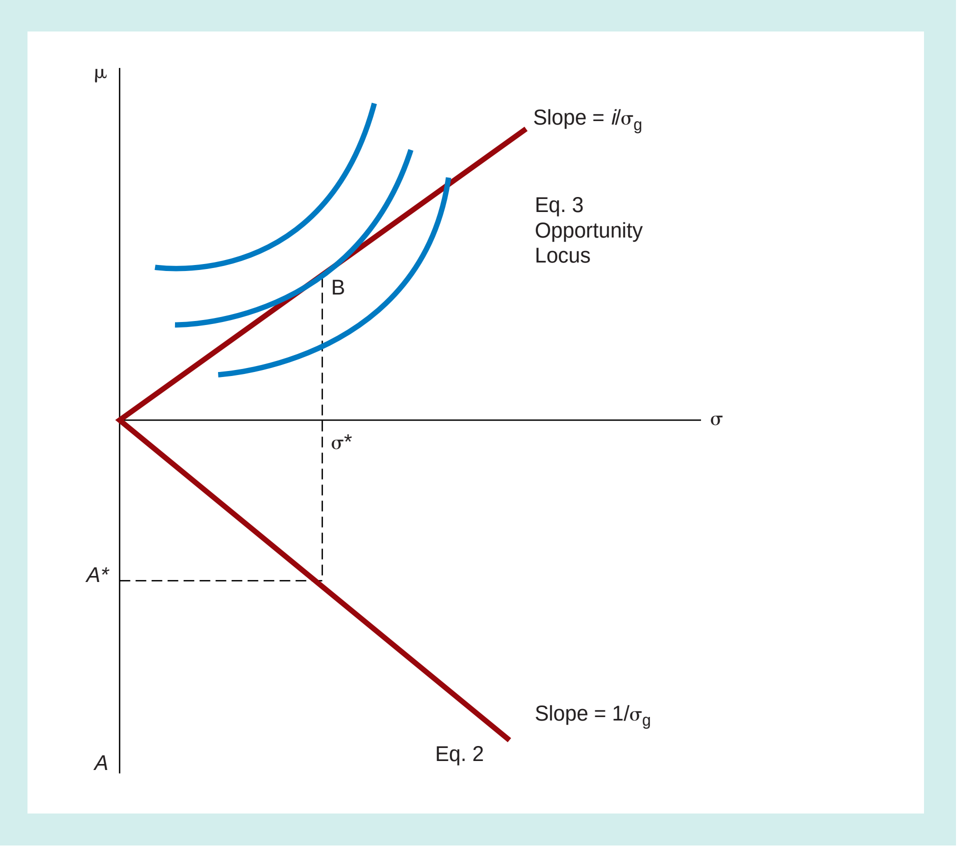
$$M^d = \frac{1}{2}\sqrt{\frac{2bT_0}{i}} = \sqrt{\frac{bT_0}{2i}} \qquad (1)$$

This is the famous *square root rule*.[2] It has these implications for the demand for money:

1. The transactions demand for money is negatively related to the interest rate i.
2. The transactions demand for money is positively related to income, but there are economies of scale in money holdings—that is, the demand for money rises less than proportionally with income. For example, if T_0 quadruples in Equation 1,

FIGURE 2 Optimal Choice of the Fraction of the Portfolio in Bonds

The highest indifference curve is reached at a point B, the tangency of the indifference curve with the opportunity locus. This point determines the optimal risk **σ***, and using Equation 2 in the bottom half of the figure, we solve for the optimal fraction of the portfolio in bonds *A**.



the demand for money only doubles.

3. A lowering of the brokerage costs due to technological improvements would decrease the demand for money.
4. There is no money illusion in the demand for money. If the price level doubles, T_0 and b will double. Equation 1 then indicates that M will double as well. Thus the demand for real money balances remains unchanged, which makes sense because neither the interest rate nor real income has changed.

[4]The indifference curves have been drawn so that the usual result is obtained that as i goes up, A^* goes up as well. However, there is a subtle issue of income versus substitution effects. If, as people get wealthier, they are willing to bear less risk, and if this income effect is larger than the substitution effect, then it is possible to get the opposite result that as i increases, A^* declines. This set of conditions is unlikely, which is why the figure is drawn so that the usual result is obtained. For a discussion of income versus substitution effects, see David Laidler, *The Demand for Money: Theories and Evidence*, 4th ed. (New York: HarperCollins, 1993).

FIGURE 3 Optimal Choice of the Fraction of the Portfolio in Bonds as the Interest Rate Rises

The interest rate on bonds rises from i_1 to i_2, rotating the opportunity locus upward. The highest indifference curve is now at point C, where it is tangent to the new opportunity locus. The optimal level of risk rises from σ_1^* to σ_2^*, and then Equation 2, in the bottom haf of the figure, shows that the optimal fraction of the portfolio in bonds rises from A_1^* to A_2^*.