time. The evidence that the interest sensitivity of the demand for money did not change from period to period also suggests that the money demand function is stable, since a changing interest sensitivity would mean that the demand for money function estimated in one period would not be used to predict that of another period.

By the early 1970s, the evidence using data from the postwar period strongly supported the stability of the money demand function when M1 was used as the definition of the money supply. For example, a well-known U.S. study by Stephen Goldfeld published in 1973 found not only that the interest sensitivity of M1 money demand did not undergo changes in the postwar period, but also that the M1 money demand function predicted extremely well throughout the postwar period.[7] Similarly, studies of the demand for money in Canada concluded that narrow money demand functions were quuite stable.[8] As a result of this evidence, the M1 money demand function became the conventional money demand function used by economists. In fact, this evidence provided the foundation for the Bank of Canada's experiment with targeting the growth rate of M1 and for its strategy of gradualism from 1975 to 1982.

The Case of the Missing Money. The stability of the demand for money, then, was a well-established fact when, starting in 1974, conventional M1 money demand functions in the United States and Canada began to severely overpredict the demand for money. Stephen Goldfeld labeled this phenomenon of instability in the demand for money function "the case of the missing money."[9] It presented a serious challenge to the usefulness of the money demand function as a tool for understanding how monetary policy affects aggregate economic activity. In addition, it had important implications for how monetary policy should be conducted. As a result, the instability of the M1 money demand function stimulated an intense search for a solution to the mystery of the missing money so that a stable money demand function could be resurrected.

The search for a stable money demand function took three directions. The first direction focused on whether an incorrect definition of money could be the reason why the demand for money function had become so unstable. As Charles Freedman and Ed Fine argue, competition between banks and near-banks, technological innovation, and high interest rates caused the payments mechanism and cash management techniques to undergo rapid changes after the beginning of money targeting in 1975.[10] This has led some researchers to suspect that the rapid pace of financial innovation has meant that the conventional definitions of the money supply no longer apply. They searched

for a stable money demand function by actually looking directly for the missing money; that is, they looked for financial instruments that have been incorrectly left out of the definition of money used in the money demand function.

Daily interest savings accounts, introduced in 1979, and daily interest chequing accounts, introduced in 1981, are one example. These accounts provided chequing privileges and paid daily interest (computed on the daily closing balance), thereby offering the small saver the opportunity to earn near-market interest rates. As a result, people found these accounts attractive and were encouraged to substitute them for demand deposits (part of M1). These accounts, however, were included in the M2 definition of the money supply and hence the demand for M1 decreased and that for M2 increased. Recent evidence using later data has cast some doubt on whether including daily interest saving and chequing accounts, and other highly liquid assets, in measures of the money supply produces money demand functions that are stable.[11]

The second direction of search for a stable money demand function was to use weighted monetary aggregates (discussed in Chapter 3). However, the results of estimating money demand functions using weighted monetary aggregates do not support the existence of a stable money demand function. For example, David Longworth and Joseph Atta-Mensah of the Bank of Canada compared the empirical performance of weighted monetary aggregates with the corresponding simple-sum aggregates and found that the theoretically superior weighted aggregates do not produce a stable money demand function.[12] This is also consistent with earlier results by John Cockerline and John Murray, also of the Bank of Canada.[13]

The third direction of search for a stable money demand function was to reevaluate the conventional money demand specifications, be looking for new variables to include in the money demand function that will make it stable. Francesco Caramaza, Doub Hostland, and Kim McPhail, for example, found that the earning-price ratio has a significant negative effect on the demand for broad money.[14] Other researchers, such as Steve Ambler and Alain Paquet, added the real stock of Canada Savings Bonds (CSB) as well as dummy variables (to capture seasonal factors and postal strikes).[15]

These attempts to produce a stable money demand function have been criticized on the grounds that the theoretical justification for including them in the money demand function is weak. Also, later research questions whether these alterations to the money demand function will lead to continuing stability in the future.[16]

Conclusion. The main conclusion from the research on the money demand function seems to be that the most likely cause of its instability is the rapid pace of financial innovation occurring after 1973, which has changed what items can be counted as money. The evidence is still somewhat tentative, however, and a truly stable and satisfactory money demand function has not yet been found. And so the search for a stable money demand function goes on.[17]

The recent instability of the money demand function calls into question whether our theories and empirical analyses are adequate.[18] It also has important implications for the way monetary policy should be conducted because it casts doubt on the usefulness of the money demand function as a tool to provide guidance to policymakers. In particular, because the money demand function has become unstable, velocity is now harder to predict, and as discussed in Chapter 21, setting rigid money supply targets in order to control aggregate spending in the economy may not be an effective way to conduct monetary policy.

The use of algebra to analyze the *ISLM* model allows us to extend the multiplier analysis in Chapter 23 and to obtain many of the results of Chapters 23 and 24 very quickly.

Basic Closed-Economy *ISLM* Model

The goods market can be described by the following equations:

Consumption function:	$C = \overline{C} + mpc\,(Y - T)$	(1)
Investment function:	$I = \overline{I} - di$	(2)
Taxes:	$T = \overline{T}$	(3)
Government spending:	$G = \overline{G}$	(4)
Goods market equilibrium condition:	$Y = Y^{ad} = C + I + G$	(5)

The money market is described by these equations:

Money demand function:	$M^d = \overline{M}^d + eY - fi$	(6)
Money supply:	$M^s = \overline{M}$	(7)
Money market equilibrium condition:	$M^d = M^s$	(8)

The uppercase terms are the variables of the model; $\overline{G}$, $\overline{T}$, and $\overline{M}$, are the values of the policy variables that are set exogenously (outside the model); and $\overline{C}$, $\overline{I}$, and $\overline{M}^d$ are autonomous components of consumer expenditure, investment spending, and money demand that are also determined exogenously (outside the model). Except for the interest rate i, the lowercase terms are the parameters, the givens of the model, and all are assumed to be positive. The definitions of these variables and parameters are as follows:

C = consumer spending
I = investment spending
$G = \overline{G}$ = government spending
Y = output
$T = \overline{T}$ = taxes
M^d = money demand
$M^s = \overline{M}$ = money supply
i = interest rate
$\overline{C}$ = autonomous consumer spending
d = interest sensitivity of investment spending
$\overline{I}$ = autonomous investment spending related to business confidence
$\overline{M}^d$ = autonomous money demand
e = income sensitivity of money demand
f = interest sensitivity of money demand
mpc = marginal propensity to consume

IS and *LM* Curves

Substituting for C, I, and G in the goods market equilibrium condition and then solving for Y, we obtain the *IS* curve:

$$Y = \frac{1}{1 - mpc}(\overline{C} + \overline{I} - mpc\,\overline{T} + \overline{G} - di) \tag{9}$$

Solving for i from Equations 6, 7, and 8, we obtain the *LM* curve:

$$i = \frac{\overline{M}^d - \overline{M} + eY}{f} \tag{10}$$

Solution of the Model

The solution to the model occurs at the intersection of the *IS* and *LM* curves, which involves solving for Y and i simultaneously, using Equations 9 and 10, as follows:

$$Y = \frac{1}{1 - mpc + de/f}\left(\overline{C} + \overline{I} - mpc\,\overline{T} + \overline{G} - \frac{d\overline{M}^d}{f} + \frac{d\overline{M}}{f}\right) \tag{11}$$

$$i = \frac{1}{f(1 - mpc) + d}[e(\overline{C} + \overline{I} - mpc\,\overline{T} + \overline{G}) + \overline{M}^d(1 - mpc) - \overline{M}(1 - mpc)] \tag{12}$$