Elementary Mathematics IN CANADA

RESEARCH SUMMARY AND CLASSROOM IMPLICATIONS

LYNDA COLGAN
Queen's University

JESSICA PEGIS

Pearson Education Canada
Addison Wesley
Toronto

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Addison Wesley

letter of welcome

Dear Educator,

Elementary Mathematics in Canada: Research Summary and Classroom Implications examines the current status of mathematics education in Canada and offers some practical ways to make a difference. The report summarizes key research findings of best practice, suggests implications for the classroom and for professional development, and helps educators with the task of defining and shaping implementation plans. Since changes in the teaching and learning of mathematics are ongoing, the report can also support further discussion and initiatives.

There is no question that mathematics education is changing: In the latest round of curriculum renewal, more attention has been paid to issues such as accountability, consistency, and rising standards. Assessment, evaluation, and reporting practices are carefully scrutinized. Resources are reviewed according to specific criteria. Professional development is expected to result in immediate and measurable differences in classroom practice and student achievement.

This change is not a single event, and the impact of innovations will be measured in years. As educators focus on moving ahead, they will need to pause and reflect on the journey so far and to think about all that has been accomplished in mathematics reform. There is no doubt that real change has begun to make a positive difference in the lives of Canadian students.

Enjoy the reading and please let us know if there is anything else we can do to support your role in mathematics education.

Dr. Lynda Colgan
Research Director

Mark Cobham
Director of Mathematics Publishing

table of contents

excecutive summary

Across Canada, individual ministries and provincial consortia have introduced curriculum policy documents inspired by the standards-based approach of the National Council of Teachers of Mathematics (NCTM). While the provincial documents differ to some extent, they are notably similar in content and rationale. In every province, there are systematic efforts to raise standards, improve student achievement, and broaden learning in three critical areas—content, processes, and skills. Likewise, every provincial curriculum document is based on a reform agenda, emphasizing teaching for conceptual understanding, problem solving, reasoning, and communication.

Elementary Mathematics in Canada: Research Summary and Classroom Implications can be used to support the implementation of new curricula across Canada. This report links what is happening in mathematics instruction to current research on best practice, and it suggests implications for further action.

The report has four overall goals:

1. **Support the development of an overall framework for implementation.** Implementation models may vary, but all models must identify clear goals, draw from a strong research base, include professional development, and provide carefully designed resources.
2. **Provide a research base for implementation of best practice.** Classroom practice should be based on research and grounded in what educators know about how students learn. In this report, the term "research" is used to denote a variety of primary and secondary sources, including books, articles and reports, curriculum documents, published standards, reviews, and interviews. Many Web links are provided for follow-up research.

3. **Establish professional development opportunities to support implementation.** In order to effect long-term changes in classroom practice, professional development must be made a priority, and it must be ongoing and sustained.

4. **Lay the foundation for future decision making related to classroom resources.** Resources can help bring research into practice at the classroom level but cannot, by themselves, achieve changes in practice; resources should complement implementation plans.

Pearson Canada will continue to work with Canadian educators in supporting implementation at the provincial, board or district, and school levels. To that end, the report will be followed up by the development of further resources and services to facilitate mathematics success in Canada.

mathematics
in Canadian schools

Historical Trends

As educators know, the call for curricular reform in North America has been all-encompassing. Many of these reforms have been influenced by the emergence of constructivism as a learning theory as well as the notion that literacy and mathematics are important foundation subjects for every citizen. In language arts, as in mathematics, the emphasis in the 1960s shifted away from memorizing facts and mastering rules and procedures to subjectively understanding the "whole" meaning of ideas, principles, processes, or procedures. The influential publications *Agenda for Action* (National Council of Teachers of Mathematics, 1985) and *Everybody Counts* (Mathematical Science Education Board, 1989) capped several decades of reform and lively discussion.

Constructivism rests on two important premises:

- Children actively construct their knowledge; they do not merely absorb the ideas to which they are exposed.
- Children assimilate new information into their existing framework of knowledge.

But the process was far from complete. Around this time, back-to-basics movements arose in Canada and the United States because some parents and educators believed that students were not being taught the crucial content and basic skills they needed to succeed. In mathematics, "basic skills" meant the memorization of algorithms and computational proficiency.

By the late 1980s, educators had refined their message, emphasizing whole meaning and basic skills as the foundation of mathematics literacy. Within the context of this discussion, in 1994 Human Resources Development Canada launched its Essential Skills Research Project, identifying "numeracy" as one of nine essential skills. Essential skills, such as reading, writing, and numeracy, are *enabling skills* required for most occupations and for many daily-life tasks. These skills also enable people to learn other skills, thereby enhancing their ability to adapt to change.[1]

Driven by the needs of a complex, technical society as well as concern over student achievement, mathematics education has continued to undergo reform. As with other disciplines, the move has been towards a standards-based curriculum that is balanced, comprehensive, and student centred. Research in the area of cognitive psychology has shown that mathematics understanding in particular arises from the ability to make strong connections and that new knowledge must be effectively connected to students' prior knowledge and experience.[2]

Societal changes compel educators to create a new curriculum.[3]

—T.F. Fennimore and M.B. Tinzmann, *What Is a Thinking Curriculum?*

A Balanced Approach

A model for a balanced approach can be gleaned by examining the notion of mathematical proficiency proposed by the US National Research Council (NRC). The NRC describes mathematical proficiency as five interrelated components, or strands:

1. Conceptual understanding—understanding mathematical ideas, operations, and relations
2. Procedural fluency—the skill to perform procedures with flexibility, accuracy, and efficiency, and to know when and how to use them appropriately
3. Strategic competence—the ability to formulate, represent, and solve mathematical problems
4. Adaptive reasoning—the ability to think logically, i.e., to think, reflect, explain, and justify
5. Productive disposition—the habitual inclination to see mathematics as worthwhile and useful, and to perceive oneself as a confident learner and doer of mathematics

The NRC has also identified several important characteristics of successful teachers of mathematics:

- Knowledge of mathematical concepts, the capacity to perform mathematical procedures with ease, and the ability to explain mathematical ideas and procedures
- Knowledge of the development of mathematical understanding in children and of the difficulties students typically encounter

- Teaching practice that takes into consideration both the curriculum and students' learning needs

One of the benchmarks of the recent mathematics reform movement was the 1989 release of the *Curriculum and Evaluation Standards for School Mathematics* by the NCTM. This document marked an early attempt by educators to articulate goals for mathematics educators and policy-makers, and to generate a broad vision for successful mathematics learning and teaching. This standards-based approach spurred curriculum change throughout North America.

Mathematics can be learned by *all* students, not just those who have a flair for it.

In 2000, the NCTM released *Principles and Standards for School Mathematics*, which expanded on the 1989 document. *Principles and Standards* reflects a new understanding about teaching and learning that has emerged in the last decade and makes detailed recommendations about what students should be learning in mathematics at the following stages:

- Pre-school and kindergarten to Grade 2
- Grades 3 to 5
- Grades 6 to 8
- Grades 9 to 12

These two documents contain several important messages. First, competency in mathematics is necessary for success in today's world. Second, mathematics can be learned by *all* students, not just those who have a flair for it. Finally, learning mathematics must go beyond the memorization of facts and steps to promote genuine conceptual understanding and facility with problem solving.

Principles and Standards for School Mathematics proposes six guiding principles for mathematics education:

1. **Equity:** All students are entitled to a coherent, challenging mathematics education that will accommodate their prior knowledge, intellectual strengths, and personal interests.
2. **Curriculum:** A curriculum should effectively organize, integrate, and present mathematical ideas in a way that supports student understanding.
3. **Teaching:** Effective mathematics instruction requires teachers to understand mathematics and to know how students learn mathematics.

4. **Learning:** Students learn mathematics through understanding. Mathematics makes sense to students when it is connected to existing knowledge in meaningful ways.

5. **Assessment:** Assessment must be an integral part of mathematics instruction and should provide useful information to both the teacher and the student.

6. **Technology:** Technologies aid the teaching, learning, and doing of mathematics.

Principles and Standards for School Mathematics also outlines what students should be able to do in mathematics. The five content and process standards shown in Table 1 are applicable to every grade level (see page 12). The content standards describe what students should know, and the process standards describe the means of acquiring the content.

The NCTM standards have become the blueprint for mathematics curriculum reform across Canada. Following the release of the 1989 document, all of Canada's provinces and territories revised their mathematics curricula to better reflect the vision articulated by the NCTM. In some provinces, a second wave of change to the mathematics curriculum has already been implemented. Each provincial curriculum is organized around the basic content categories shown in Table 1, although the Western Canadian Protocol (WCP) and the Atlantic Provinces Education Foundation (APEF) have included Measurement as a subcategory of Shape and Space. Table 2 summarizes the provincial strands (see page 12).

The NCTM process standards are also reflected in the different provincial curricula, as shown in Table 3 on page 12. Local curriculum writers have not only enthusiastically accepted the vision of the NCTM content and process standards, they have also built on it and made it their own:

When students learn mathematics, they do more than master basic skills; they acquire a concise and powerful means of communication. Knowledge of mathematical language, structures, and operations will help students to reason, to justify their conclusions, and to express ideas clearly. Students also need to be able to use mathematics in connection with technology and in their daily lives and, eventually, in the workplace.

— *The Ontario Curriculum, Grades 1–8, Mathematics*, page 5

Table 1 NCTM Standards

Content Standards	Process Standards
Number and Operation	Problem Solving
Patterns, Function, and Algebra	Reasoning and Proof
Geometry and Spatial Sense	Communication
Measurement	Connections
Data Analysis, Statistics, Probability	Representation

Table 2 Provincial Strands

NCTM Content Standards	WCP Strands	Ontario Strands	APEF Strands
Number and Operation Patterns, Function, and Algebra Geometry and Spatial Sense Measurement Data Analysis, Statistics, Probability	Number Concepts Operations Patterns and Relations Shape and Space Statistics and Probability	Number Sense and Numeration Patterning and Algebra Geometry and Spatial Sense Measurement Data Management and Probability	Number Concepts, Number Relationships, Operations, and Number Sense Patterns and Algebra Shape and Space Data Management and Probability

Table 3 Provincial Curricula

NCTM Process Standards	WCP Mathematical Processes	Ontario Knowledge and Skills	APEF Unifying Ideas
Problem Solving Reasoning and Proof Communication Connections Representation	Communication Connections Estimation and Mental Mathematics Problem Solving Reasoning Technology Visualization	Problem Solving Understanding of Concepts Application of Mathematical Procedures Communication	Problem Solving Communication Reasoning Connections

The expectations related to students' problem-solving skills are … stringent. Students will be expected to describe what they are doing in mathematics and to explain why they are doing it.

— *The Ontario Curriculum, Grades 1–8, Mathematics*, page 3

Mathematics is a common human activity, increasing in importance in a rapidly advancing, technological society. A greater proficiency in using mathematics increases opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions, and actively create new knowledge in striving for self-fulfillment.

— *The Common Curriculum Framework, K–12 Mathematics*, Western Canadian Protocol, page 2

Mathematics learning is an active and constructive process; learners are individuals who bring a wide range of prior knowledge and experiences, and who learn in various styles and at different rates; learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking….

— *Atlantic Canada Mathematics Curriculum, Primary to 3*, page 8

Testing in Elementary Mathematics

The results of international, national, and provincial testing suggest that curriculum reform has had an impact on overall mathematics proficiency among Canadian students.

International Testing: TIMSS

In 1995, the Third International Mathematics and Science Study (TIMSS) examined mathematics and science achievement among elementary students in 42 countries, including Canada. A repeat study conducted in 1999 (TIMSS-R) collected more data on Grade 8 students in 38 countries. Thousands of Canadian students have participated in both studies, providing a comparison of performance levels between 1995 and 1999.

The mathematics portion of the TIMSS examined the following five content areas, using three types of questions—multiple choice, short answer, and extended response:

In 1999, Canadian students did as well or better than students from 30 countries.

- Fractions and number sense
- Measurement
- Data representation, analysis, and probability
- Geometry
- Algebra

In 1995, Canadian Grade 4 students scored significantly below the international average in mathematics. By 1999, those same students—now in Grade 8—were scoring higher than the international average. In 1999, Canadian students did as well or better than students from 30 countries. Ten countries scored above Canada. Only six countries scored above British Columbia—Singapore, Korea, Hong Kong, Japan, Belgium, and the Czech Republic. While it is impossible to attribute these improved scores to one factor, the implementation of new curricula emphasizing reasoning, problem solving, and metacognition has been noted.[4]

National Testing: SAIP

The School Achievement Indicators Program (SAIP) is a national program that regularly assesses student achievement in three areas—reading and writing, science, and mathematics. Overseen by the Council of Ministers of Education of Canada, the program examines the performance levels of 13- and 16-year-old students. The goal is to provide a comprehensive picture of student achievement across the country.

In mathematics, students are tested on content and problem solving. Achievement is expressed in levels representing a continuum of mathematics knowledge and problem-solving skills acquired over time. For example, Level 1 represents the most basic level of mathematical knowledge and skill typical of a young elementary student; Level 5 represents the knowledge and skill base of a student who has completed all specialized mathematics course work at the secondary level.

SAIP testing in 2001 showed the following results:

- Approximately two-thirds of Canadian 13-year-olds reached the target level for this age group—Level 2 or higher in mathematics content and problem solving.
- One quarter of the 13-year-olds reached Level 3 or higher.

- Almost half of the 16-year-olds reached the target level for this age group—Level 3 in mathematics content and problem solving.
- Both age groups showed improvement in the areas of content and problem solving when compared with the 1997 test results.

Following the 2001 assessment, a panel of 100 representatives from across Canada was convened to review the assessment instruments and student results in order to answer the following question: "What percentage of Canadian students should achieve at or above each of the five performance levels?" The panel concluded that despite the gains over four years, students in both age groups still fell short of the levels to be expected in both content and problem solving.[5]

Provincial Testing

Province-wide achievement testing of elementary and high-school students now occurs across Canada. The provinces use a variety of assessment methods and approaches to collect information about their students. Most of these assessments are intended to improve accountability. The tests examine student achievement in core subject areas at different grade levels and determine if students have met the knowledge and skill requirements of the provincial curriculum at each level. Such assessments also help to identify areas requiring extra instruction or intervention. All results are reported publicly.

Mathematics Highlights

The range of results obtained in these assessments indicates that the process of curriculum implementation is far from finished and is undoubtedly affected by a range of factors. Note the following results:

British Columbia In 2001, the British Columbia Foundation Skills Assessment reported that 84 percent of Grade 4 students met or exceeded the expectations defined for their grade level. At the Grade 10 level, 74 percent of students met or exceeded the expectations defined for their grade. It was also reported that 23 percent of the work of Grade 10 students did not meet the expectations defined for that grade.[6]

Alberta In 2001, 88.5 percent of Grade 3 students met the "acceptable standard" for knowledge, and 86.3 percent met the acceptable standard for skills, based on learning outcomes expected at that grade level. Results for Grade 6 students were similar, with 86.5 percent of students meeting the acceptable standard in knowledge and 84.7 percent meeting the acceptable standard in skills.[7]

Ontario In 2000–2001, 61 percent of Grade 3 students and 54 percent of Grade 6 students achieved a Level 3. These scores represented a dramatic improvement over the 1997–1998 test results, especially at the Grade 3 level.[8]

Nova Scotia In 2001, Grade 5 students were assessed on work covered in Grades 3 to 5. The average score was only 42 percent (pass = 50 percent). Students achieved the highest score—52 percent—in computational fluency.[9]

Endnotes

1. Human Resources Development Canada (1994), *Introduction to occupational profiles*, What are essential skills? <www15.hrdc-drhc.gc.ca/english/intro.asp>
2. National Research Council (2001), J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Adding it up: Helping children learn mathematics*, Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, Washington, DC: National Academy Press. For extensive research into the area of mathematics learning, see page 118, footnote 4.
3. T.F. Fennimore & M.B. Tinzmann (1990), *What is a thinking curriculum?* North Central Regional Educational Laboratory. To view the entire article, go to <www.ncrel.org/sdrs/areas/rpl_esys/thinking.htm>.
4. D.F. Robitaille, A.R. Taylor, & G. Orpwood (1999), *TIMSS-Canada report, volume 1: grade 8*. Executive Summary available at <www.curricstudies.educ.ubc.ca/wprojects/TIMSS/TIMSSPop2.pdf>.
5. School Achievement Indicators' Program, *Report on mathematics III assessment, 2001: Highlights*. <www.cmec.ca/saip/math2001/public/highlights.en.pdf>
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9. Nova Scotia Ministry of Education (2001), *Minister's report to parents: Student assessment results for Nova Scotia*. <ftp://ftp.ednet.ns.ca/pub/educ/reports/ns_assessments.pdf>

what educators have told us

Two years ago, Pearson Canada embarked on a research journey. Our goal was both simple and daunting—to understand the challenges of teaching mathematics in Canada in an era of curriculum reform.

Research can take many forms. In this report, we summarize important literature and policy in the field of mathematics education, including the results of mathematics assessments and theories of learning and teaching. However, the report does not end there. The issues presented and many of the recommendations emerging from those issues belong to the educators who talked to us about the challenges of teaching mathematics. Since 2000, this thoughtful, qualitative information has enriched our understanding of educator perspectives and remains integral to our research.

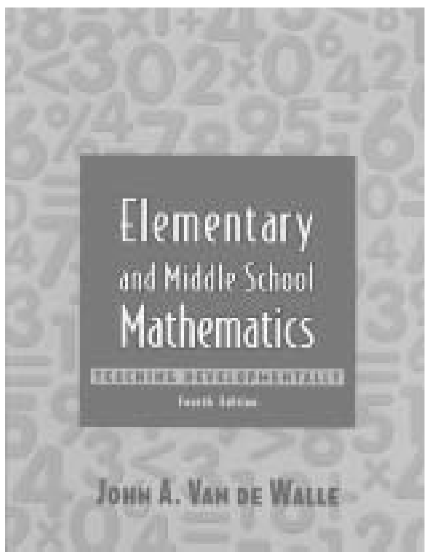
Pearson Canada began its field research two years ago by examining the academic literature on children's learning of mathematics. Next, we visited Canadian classrooms across the country to conduct focus groups and one-on-one interviews with teachers. In all, more than 1000 teachers participated in our research process. This process included focus groups, one-on-one interviews, a national survey, and field testing. We asked teachers and principals to discuss the challenges of implementing a new curriculum, to state their "key issues," to describe successful professional development, and to critique and recommend current resources.

In all, more than 1000 teachers participated in our research process. This process included focus groups, one-on-one interviews, a national survey, and field testing.

In 2001, Pearson began working with mathematics educators to consolidate a picture of mathematics education and to answer such questions as, "Where are we now?" "Where do we want to go?" and "How do we get there?" Our research expanded to include interviews with consultants, parents, and members of professional mathematics associations.

Teachers in particular have informed the research process, and their needs have become the impetus for further research and publishing initiatives. Interviews show that while teachers are enthusiastic about mathematics reform, they want support in specific areas:

- teacher knowledge of mathematics content
- teaching developmentally
- instruction and assessment strategies
- teaching through problem solving
- teaching the "Big Ideas" (see caption below)
- managing a mathematics program
- communicating with parents



"Big Ideas" are large mathematical concepts, e.g., "Addition and subtraction are connected. Addition names the whole in terms of the parts, and subtraction names a missing part." These concepts are discussed by John A. Van de Walle in his text *Elementary and Middle School Mathematics: Teaching Developmentally.*

Interviews revealed that professional development should be deeply connected to the program of study and must be "ongoing, customized and immediate."

Professional development is another key issue identified by educators. Nationally, both teachers and principals who responded to our surveys ranked professional development as highly important. Teachers also articulated concern about the success of their students, intuiting that their own confidence in the mathematics classroom is linked to student achievement. They "feel responsible" for the curriculum and are calling for better support and resources to implement it effectively.

research issues

The next section of this report revisits the issues that educators identified as most significant during Pearson's research-gathering phase:

Numeracy

What Research Tells Us

As discussed earlier, numeracy is one of the nine essential skills identified by Human Resources Development Canada (see Mathematics in Canadian Schools, Historical Trends, page 8). The importance of numeracy is also emphasized by the National Council of Teachers of Mathematics. In its *Principles and Standards for School Mathematics*, the NCTM identifies lifelong proficiency in mathematics as a "need" in a changing world:

> The underpinnings of everyday life are increasingly mathematical and technological. For instance, making purchasing decisions, choosing insurance and health plans, and voting knowledgeably all call for quantitative sophistication.[1]

The British Columbia Ministry of Education defines "numeracy" as follows:

> Numeracy refers to the application of mathematical understanding in daily activities at school, at home, at work, and in the community. It involves both using mathematical skills and knowing how mathematics can be used to solve problems.… Numerate individuals not only "know" mathematics, but understand it in personally meaningful terms.[2]

Throughout Canada, educators are using numeracy in many cross-curricular applications by encouraging students to

- recognize the "math" in varied tasks;
- structure tasks into logical steps;
- record and calculate on a daily basis (e.g., simple budgeting);
- talk about various procedures and methods of solving problems; or
- explain visual representations of mathematics concepts.

What Are the Implications?

- All students benefit from opportunities to develop and apply numeracy skills from early childhood.
- Mathematics should be taught beyond the classroom, within the context of a variety of disciplines and real-life tasks.
- Numeracy is necessary for lifelong learning and career planning.
- Society must recognize numeracy as well as literacy as an essential skill.

Society must recognize numeracy as well as literacy as an essential skill.

Endnotes

1. National Council of Teachers of Mathematics (2000), *Principles and standards for school mathematics*, p. 4.
2. British Columbia Ministry of Education (2002), *B.C. performance standards: Numeracy*, p. 9.

For Further Reading

Arthur Sheen, L. (Ed.). (2001). *Mathematics and democracy: the case for quantitative literacy*. National Council on Education and the Disciplines, 2001. Paperbound. To see a review of this title, visit <www.maa.org/reviews/mathdemo.html>.

McCain, M. & Mustard, F. (1999). *The early years study: reversing the real brain drain*. Report delivered to the premier of the province and to the minister responsible for children. To view this report, visit <www.founders.net/ey/home.nsf/home!openpage> and click on the left-hand menu.

Student Achievement

What Research Tells Us

Canada is part of a worldwide movement to raise the achievement levels of students and to implement national standards for assessing student performance. This reform has been motivated by concern about student achievement in key learning areas such as literacy and mathematics and by the need for students to compete in a highly competitive global economy. Nations such as the United States, Britain, Australia, New Zealand, and South Africa are moving towards standards or benchmarks to measure student achievement.

A standards-based system has several important features. First, the system establishes specific standards that articulate performance objectives and allow for rigorous content-based assessment. Second, it relies on the regular testing of students to determine whether these standards are being met, and it reports those results publicly. In Canada, yearly results from national, provincial, and district large-scale assessments are made available to teachers, administrators, parents, and interested members of the public.

How this information is used speaks directly to the third feature of a standards-based system: information about student achievement must be used to help educators and students meet performance goals through improvement programming, targeted assistance, or curriculum reform. Recently, the Education Quality and Accountability Office (EQAO) launched a new initiative, the Education Quality Indicators Program (EQUIP), which will collect additional information on factors inside and outside the school community that affect student achievement. This information will also provide a broader framework for interpreting test scores on national and provincial assessments.[2]

Finally, a standards-based system necessitates the alignment of curriculum, assessment, classroom instruction, learning resources, and professional development.

Whether standards endure or not, what will remain is the demand for accountability.... Whether we focus on standards or not, we're entering an era of accountability that has been created by technology and the information explosion.[1]

— Robert J. Marzano, senior fellow at Mid-continent Research for Education and Learning

What Are the Implications?

- Student learning goals and targets for student achievement must be defined and commonly understood.
- Success in education must be a priority for all learning partners—students, teachers, and parents—so that students reach their learning goals.
- Curriculum, instruction, resources, assessment, and professional development must be aligned to ensure student success.

Student learning goals and targets for student achievement must be defined and commonly understood.

Endnotes

1. Marge Scherer (September 2001), How and why standards can improve student achievement—a conversation with Robert J. Marzano, *Educational Leadership, 59*(1). For the complete interview, go to <www.ascd.org/readingroom/edlead/0109/scherer.html>.
2. For more information on EQUIP, visit the EQAO Web site at <www.eqao.com/eqao/home_page/03e/3_1e.html>.

In a national survey, respondents ranked "curriculum match" as the most important reason for considering the purchase of a new resource.

Meeting the Curriculum

What Research Tells Us

According to research conducted by Pearson Canada, teachers and administrators believe that the ideal mathematics program must provide a 100 percent curriculum match. In a national survey, respondents ranked "curriculum match" as the most important reason for considering the purchase of a new resource.

It is widely recognized that coherent resources organize or cluster curriculum expectations in a way that reflects developmental progression within and across grades. The NCTM describes a coherent program as one that "effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on, or connect with, other ideas, thus enabling them to develop new understandings and skills." It also notes that "sequencing lessons coherently across units and school years is challenging."[1]

Learning materials such as textbooks have a significant role to play in meeting this challenge. In a survey of Canadian teachers that examined the role of textbooks on teacher practice, curriculum expert John Ross and his colleagues noted that "textbooks … aligned with provincial and district curriculum policy contribute to implementation of reform."[2]

The following teacher recommendations emerged from focus groups and one-on-one interviews conducted by Pearson Canada. These recommendations indicate that mathematics programs and resources can relate to the curriculum in a dynamic fashion by

- including a roadmap for teachers, with direct curriculum links and references;
- offering an outcomes/expectations-based approach with teacher strategies for building concepts;
- clustering the outcomes/expectations around the Big Ideas;
- modelling specific problem-solving strategies and skills; and
- making topics relevant through real-life examples and situations.

What Are the Implications?

- Programs and resources must be aligned in content and organization with the curriculum outcomes/expectations.
- Resources must support teaching by being easily accessible and user-friendly.
- Resources must cluster outcomes/expectations around the Big Ideas, present strategies for learning and teaching, and model skills.
- The curriculum should be brought to life through relevant, engaging examples and problems.

The curriculum must be brought to life through relevant, engaging examples and problems.

Endnotes

1. National Council of Teachers of Mathematics (2000), *Principles and standards for school mathematics*, p. 4.
2. J. Ross, D. McDougall, & A. Le Sage (2001), The contribution of textbooks to the implementation of reform: The case of Quest 2000. Chapter to appear in *OISE papers in mathematics education*, edited by D. McDougall.

Computational Fluency

What Research Tells Us

The real-life advantages of computational fluency are numerous, for example, easily estimating a tip or considering price and volume to determine whether one brand offers better value than another.

The National Research Council refers to confidence and competence with numbers as "procedural fluency"—knowing the steps and rules for calculating and computing, knowing when and how to use them, and performing them with accuracy, efficiency, and flexibility. "Without sufficient procedural fluency, students have trouble deepening their understanding of mathematical ideas or solving mathematical problems."[1]

Computational fluency involves more than performing the calculations of addition, subtraction, multiplication, and division, although a foundation in these operations is critical. Children also require "number sense." Number sense refers to flexible thinking and intuitive ideas about numbers such as

- having a sense of the relative size of numbers, e.g., "larger than," "smaller than," and "about the same size as";
- being able to estimate numbers accurately to facilitate computation, for example, rounding up numbers to the nearest 10 before adding or subtracting, and then subtracting the difference;
- having strategies for multiplying a number by any multiple of 10 or power of 10;
- understanding the effects of operating with large numbers such as 1000 or higher;
- being able to connect numbers to real-world concepts, such as estimating how many people can fit into an elevator compared to how many people attend a Canada Day parade; or
- understanding the probability that a particular event will occur. [2]

Number sense refers to flexible thinking and intuitive ideas about numbers.

It is noteworthy that all computational strategies depend on understanding place value. The National Research Council states the following:

> A good conceptual understanding of place value in the base-10 system supports the development of fluency in multi-digit computation. Such understanding also supports simplified but accurate mental arithmetic and more flexible ways of dealing with numbers than many students ultimately achieve.[3]

What Are the Implications?

- Students' strategies for performing calculations should be flexible, accurate, and efficient.
- Students must learn strategies for mental mathematics, such as counting-on and decomposition.
- Students should be encouraged to think about numbers in real-life situations as well as school contexts.
- Students should learn that there are a variety of ways to solve problems, including using standard algorithms.
- Elementary-school mathematics education should focus on the key constructs of counting, number relationships, grouping, and partitioning.

Students should be encouraged to think about numbers in real-life situations as well as school contexts.

Endnotes

1. National Research Council (2001), J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Adding it up: Helping children learn mathematics*, Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, Washington, DC: National Academy Press, p. 122.
2. Ideas in this section are drawn from the following sources: *Adding it up: Helping children learn mathematics*, pp. 121–123; J.A. Van de Walle (2001), *Elementary and middle school mathematics: Teaching Developmentally*, 4th ed., Addison Wesley Longman, Inc., chapters 9 and 11.
3. National Research Council (2001), *Adding it up: Helping children learn mathematics*, p. 121.

Teaching Developmentally

What Research Tells Us

Effective teaching of mathematics requires knowing how children learn and how their mathematical thinking develops. Educators and psychologists such as Jerome Bruner, Jean Piaget, Lev Vygotsky, and John Dewey have had a significant impact on our understanding of this process. Their theories have helped to shape both the classroom environment and classroom instruction.[1]

Piaget was the first to propose the idea of *mental structures*, which determine how human beings incorporate new information. When new information makes sense to our existing mental structures, that information is "bent" to fit our understanding—*accommodation* in Piaget's terms. On the other hand, when a new concept differs greatly from our existing mental structures, we see no need to incorporate it. New information is either *rejected* or *assimilated* so that it fits into our mental structures.

How do these theories apply to the teaching of mathematics? They strongly suggest that children learn by assimilating new information into their pre-existing ideas, and they modify their understanding in light of new data. Over time, their ideas gradually increase in complexity and power.

Learning involves a complex series of external and internal events that result in the creation of mental connections. The teacher's role is to facilitate the student's own cognitive processes by carefully selecting tasks that elicit thinking just beyond the student's current state of knowledge. This strategy will result in significant advancement in the student's knowledge and use of strategies. Classroom instruction can stimulate learning by relating new material to old and by using oral and written activities to help students make intellectual connections among new ideas.

If educators try to teach concepts that are beyond the developmental stage of their students, no amount of teaching and external reinforcement will result in learning.

What Are the Implications?

- Teachers must have the opportunity to receive training in the process of mathematical development to help advance student understanding.
- Lessons should give students explicit opportunities to integrate new material into their existing base of ideas.
- Students benefit from working with new but related mathematical situations.
- Authentic tasks set in a meaningful context are the best way to introduce new knowledge and skills.

Understanding developmental stages clearly provides a framework for planning effective mathematics lessons to meet the needs of all students.

Endnote

1. J. Bruner (1966a), *Studies in cognitive growth: A collaboration at the Center for Cognitive Studies*, New York: Wiley & Sons; J. Bruner (1966b), *Toward a theory of instruction*, Cambridge, MA: Harvard University Press; J. Piaget (1972), *The psychology of the child*, New York: Basic Books; J. Piaget (1990), *The child's conception of the world*, New York: Littlefield Adams; L. Vygotsky & S. Vygotsky (1980), *Mind in society: The development of higher psychological processes*, Cambridge: Harvard University Press; L. Vygotsky (1986), *Thought and language*, Boston: MIT Press; J. Dewey (1997a), *Experience and education*, New York: MacMillan Publishing Company; J. Dewey (1997b), *How we think*, New York: Dover Publications.

Conceptual Understanding

> The tools we use to build understanding are our existing ideas, the knowledge that we already possess.
>
> — John A. Van De Walle

What Research Tells Us

Mathematics reform has drawn a great deal from the work of contemporary scientists such as Ernst von Glasersfeld and Leslie P. Steffe, whose theories about children's mathematical thinking have paved the way for new classroom standards and new definitions of "learning."[1] By studying how young children reason and think mathematically, these scientists propose—like their forerunners—that when new knowledge is acquired, it must fit into an existing base of knowledge. Students with deep mathematical understanding have "organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know."[2]

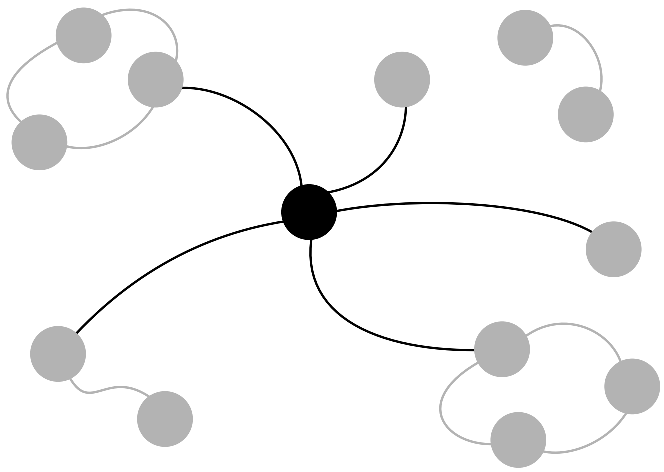
In *Elementary and Middle School Mathematics*, John Van de Walle defines conceptual knowledge of mathematics as logically interconnected ideas developed over time. Students and teachers who have a conceptual understanding of mathematics truly understand it—they have more than just a grasp of its facts, rules, and procedures.

Learning mathematics "with understanding" is also one of the articulated principles in NCTM's *Principles and Standards for School Mathematics*, which notes that "learning without understanding has been a persistent problem since at least the 1930s.…"[3]

A conceptual understanding of mathematics is significant in many respects:

- Children have an intuitive understanding of mathematics, which can be developed through tasks that connect new knowledge to existing knowledge and through opportunities to examine different strategies for problem solving. In the classroom, teachers can support "the recognition of connections among ideas and the reorganization of knowledge."[4] In Van de Walle's diagram in Figure A, the grey dots represent existing ideas that connect to construct a new idea, which is represented by the black dot.

Figure A New ideas are connected to old.



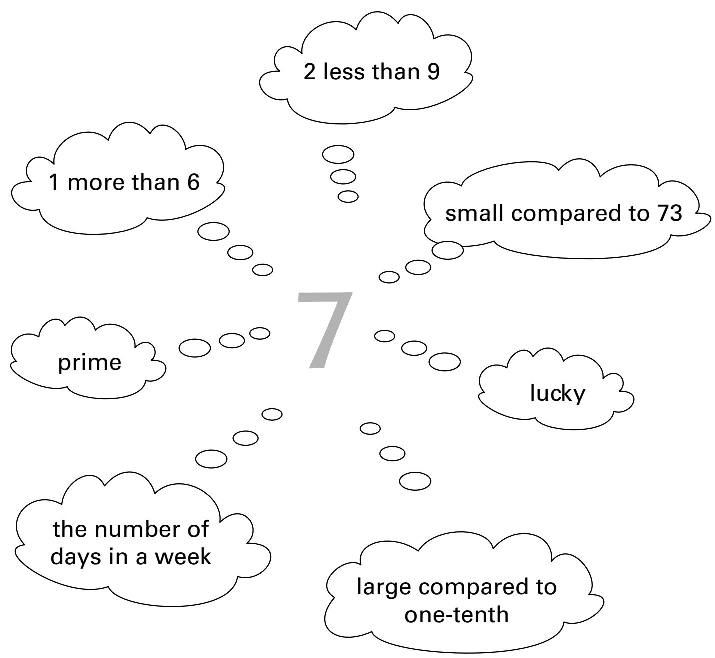
- Mathematical procedures learned with understanding are easily recalled and applied in a variety of settings. For example, while many students can initially master the rules for fractions, they may forget the rules quickly if they do not understand them.
- Conceptual understanding supports the retention of knowledge. Because this knowledge is connected to other knowledge, it is retained and readily accessible.
- Conceptual understanding provides the basis for acquiring new knowledge and solving unfamiliar problems. When students understand concepts, they may be able to apply them in unfamiliar situations.
- Conceptual understanding enables students to spot and correct errors they have made in problem solving.
- Because conceptual understanding involves the "clustering" of knowledge principles, students have less to remember (see Figure B on page 32). This idea is explained in *Adding It Up: Helping Children Learn Mathematics:*

Conceptual understanding frequently results in students having less to learn because they can see the deeper similarities between superficially unrelated situations. Their understanding has been

When students understand concepts, they may be able to apply them in unfamiliar situations.

encapsulated into compact clusters of interrelated facts and principles. The contents of a given cluster may be summarized by a short sentence or phrase like "properties of multiplication," which is sufficient for use in many situations. If necessary, however, the cluster can be unpacked if the student needs to explain a principle, wants to reflect on a concept, or is learning new ideas.[5]

Figure B The idea of "7" as constructed by a young child. Here, the idea clusters are readily evident, and the web of connections can continue to expand.



What Are the Implications?

- Assessment must value more than the "right" answer.
- Students learn mathematics by *doing* mathematics.
- Students must learn to make connections between different ideas and representations of concepts.
- Teachers require both mathematics knowledge and pedagogical skill to facilitate conceptual understanding in their students.
- Textbooks and other resources must give concrete examples of how to cluster ideas to represent concepts.

Students learn mathematics by *doing* mathematics.

Endnotes

1. E. von Glasersfeld (1987), *The construction of knowledge: Contributions to conceptual semantics*, Seaside, California: Intersystems Publications; E. von Glasersfeld (1995), *Radical constructivism: A way of knowing and learning*, London and Washington: The Falmer Press; L.P. Steffe (1996), *Theories of mathematical learning*, Lawrence Erlbaum Associates.
2. National Research Council (2001), *Adding it up: Helping children learn mathematics*, J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press, p. 118.
3. National Council of Teachers of Mathematics (2000), *Principles and standards for school mathematics*, p. 20.
4. Ibid., p. 21.
5. National Research Council (2001), *Adding it up: Helping children learn mathematics*, p. 120.

Problem Solving

What Research Tells Us

A key issue in mathematics reform has been the emphasis on learning skills, rules, and procedures in the context of problem solving. As part of its comprehensive recommendations regarding proficiency in mathematics, the National Research Council has stated that problem solving should be viewed as an important context for learning:

> Problem-solving ability is enhanced when students have opportunities to solve problems themselves and to see problems being solved. Further, problem solving can provide the site for learning new concepts and for practicing learned skills. We believe that problem solving is vital because it calls on all strands of proficiency, thus increasing the chances of students integrating them. Problem solving also provides opportunities for teachers to assess students' performance on all of the strands.[2]

Many of these ideas are captured in Chapter 4 of John A. Van de Walle's *Early and Middle School Mathematics*, "Teaching Through Problem Solving." In this chapter, Van de Walle identifies five reasons for using a problem-solving approach to teaching mathematics. He defines a "problem" as a task or exploration for which the solution has not been explained. The problem begins with the ideas that students already have; it challenges students mathematically and makes them responsible for justifying or explaining their answers or methods.

The opportunity to solve problems

- focuses the students' attention on making sense of mathematical ideas;
- develops "mathematical power" by engaging most of the process standards;
- encourages students to believe that they are capable of doing mathematics and that mathematics make sense;
- provides ongoing data for assessment; and
- engages students, stimulating them to continue problem solving.

> Solving problems is not only a goal of learning mathematics but also a major means of doing so.... Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of a mathematics program.[1]
>
> — National Council of Teachers of Mathematics

Van de Walle also states that when students solve problems regularly, they want to solve *more* problems and strive to develop other methods for attacking them. In other words, as their knowledge of procedures becomes more grounded in understanding, their confidence as problem-solvers grows.[3]

The opportunity to solve problems encourages students to believe that they are capable of doing mathematics and that mathematics make sense.

What Are the Implications?

- Students must be exposed to age-appropriate problems from the earliest point of their mathematical learning.
- Students must learn specific strategies for solving problems.
- Students benefit when non-routine problems are posed on certain tests and assignments. When students have sufficient skill to apply procedures, non-routine problems become routine and expected.
- Problems should be set in contexts that are inherently interesting to children.

Endnotes

1. National Council of Teachers of Mathematics (2000), *Principles and standards for school mathematics*, p. 52.
2. National Research Council (2001), *Adding it up: Helping children learn mathematics*, J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, Washington, DC: National Academy Press, p. 420.
3. These ideas are drawn from of J.A. Van de Walle (2001), *Elementary and middle school mathematics: Teaching developmentally*, 4th ed., Addison Wesley Longman, Inc., pp. 41–42.

Reasoning

What Research Tells Us

The hallmark of reasoning is the ability to explain and justify one's thinking.

Mathematics reasoning in the elementary grades focuses on the ability to think logically about relationships between concepts and situations, to reflect, and to explain and justify student work. A student uses reasoning to navigate the facts, procedures, concepts, and techniques of mathematics and to fit them together in a sensible way. The hallmark of reasoning is the ability to explain and justify one's thinking.

To hone their reasoning skills, students must draw on other kinds of mathematical proficiency. For instance, they use their strategic competence to figure out what the problem is (or how to represent it), decide what strategy to use to solve it, identify relationships in the problem, understand the difference between relevant and irrelevant information, consider connections and alternatives, observe patterns, conjecture, estimate, validate their answers, and generate alternative plans whenever the current plan seems ineffective. Students also use conceptual understanding to provide metaphors and representations that can help them make sense of the solution in a personal way. While carrying out a solution, learners apply their knowledge of algorithms and procedures to monitor their progress and to self-assess their answers.[1]

In Canadian curricula, mathematics reasoning is highlighted as a critical process skill. The Alberta Mathematics Program of Studies, K–6, states the following: "Students need to develop confidence in their ability to reason and to justify their thinking within and outside of mathematics."[2]

The NCTM states that "being able to reason is essential to understanding mathematics. Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts." Consider the following contexts:

Students recognize that reasoning is a fundamental aspect of mathematics. Reasoning can be nurtured at a very early age by asking students to explain and justify their observations with questions such as, "Why do you think that's true?" and helping students distinguish between real evidence and non-evidence.

Students make and investigate mathematical conjectures.
Conjectures are opinions that lack evidence—for now. One of the values of conjecturing is that it leads to discovery. Teachers can encourage students to conjecture by posing the right questions such as, "What do you think will happen next?"

Students develop and evaluate mathematical arguments.
In the early grades, students tend to justify their responses by referring to a specific case and reasoning "from what they know." As students mature, they should be able to move beyond the results of a particular problem and generalize to other situations.

Students use various types of reasoning.
Students vary their reasoning when three conditions are met: they have a sufficient knowledge base; the task is understandable and motivating; and the context is familiar and comfortable.

The shrewd guess, the fertile hypothesis, the courageous leap to a tentative conclusion—these are the most valuable coin of the thinker at work. But in most schools guessing is heavily penalized and is associated somehow with laziness.

— Jerome S. Bruner, *The Process of Education*

What Are the Implications?

- Reasoning for early elementary students is driven by teacher inquiry, i.e., posing questions such as, "What pattern do you see here?"
- Students benefit from opportunities to conjecture, to solve problems, to identify the concepts and procedures involved, to explain and justify their solutions to others, and to pose their own problems.
- In explaining and justifying solutions, students should be encouraged to relate the concepts and procedures to what they already understand.

Endnotes

1. For a discussion of adaptive reasoning and strategic competence, see National Research Council (2001), *Adding it up: Helping children learn mathematics*, J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, Washington, DC: National Academy Press, pp. 124–131.
2. Alberta Learning (1997), *Introduction to the Alberta mathematics program of studies, kindergarten to grade 6*, p. 8.

Communication

What Research Tells Us

Communication in mathematics involves more than written and symbolic notation. It includes discussing and describing "mathematical thinking" and interpreting and evaluating what other students say.

All provinces have recognized communication as a category of achievement, and many provincial tests assess communication as a specific skill. Canadian students are being asked to describe their understanding of mathematical concepts, to talk about the procedures they use, to justify their problem solving, and to make connections between different representations of number (for example, relate the symbol of a number to a graphic presentation of a number).

Classroom communication is also important because students must learn to validate—and evaluate—answers by asking such questions as, "Is this the right technique to use here?" "When would this technique be more advantageous?" Traditionally, discussion has not been a part of mathematics class; now it is centre stage.

Maintaining that successfully *communicating* about mathematics is part of the process of *learning* mathematics, the NCTM articulates the following principles:

- Communication helps students organize and reflect on their own mathematical thinking. For instance, when students ask questions about a puzzling concept or describe the steps they took to solve a problem, they are encouraged to think out loud in an organized way. This process can clarify ideas and resolve misconceptions.
- Communication is necessary to present mathematical ideas to the larger community, and building community in the classroom is key to attaining this goal. Students should feel that their ideas are valued and that it is safe to express them. In addition, students gain new insights by viewing ideas from perspectives other than their own. The NCTM states the following:

Communication helps students organize and reflect on their own mathematical thinking.... This process can clarify ideas and resolve misconceptions.

- In order for a mathematical result to be recognized as correct, the proposed proof must be accepted by the community of professional mathematicians. Students need opportunities to test their ideas on the basis of shared knowledge in the mathematical community of the classroom to see whether they can be understood and if they are sufficiently convincing.[1]

- Communication of mathematical ideas and strategies allows others to analyze and evaluate them. Students benefit from group situations where they can hear and critique other students' strategies for solving problems and reflect on the strengths and weaknesses of these strategies.

Developing a mathematics vocabulary is also part of being able to communicate ideas precisely. For example, students should begin using specific terminology such as "translations, rotations, and reflections" in the early grades and should continue using formal vocabulary throughout high school.

Students should feel that their ideas are valued and that it is safe to express them.

What Are the Implications?

- Students benefit when "talking mathematics" in the classroom is valued and routine.
- Teachers can model mathematical thinking by using strategies such as "think aloud."
- Students should be encouraged to use mathematics terminology and conventions appropriately.
- Students learn from seeing examples of solutions that integrate multiple representations.
- Mathematics dictionaries, illustrated mathematics glossaries, and other visual aids are useful tools to have in the classroom.

Endnote

1. National Council of Teachers of Mathematics (2000), *Principles and standards for school mathematics*, p. 61.

Assessment and Evaluation

> Assessment for learning is the process of seeking and interpreting evidence for use by learners and their teachers to decide where the learners are in their learning, where they need to go and how best to get there.[1]
>
> — Assessment Reform Group

What Research Tells Us

In education, effective assessment and evaluation has many functions, such as

- providing feedback to students, teachers, and parents;
- identifying and selecting students for particular programs;
- certifying student accomplishment;
- demonstrating the accountability of schools, districts, and systems; and
- evaluating particular programs, techniques, or resources.

Current research, policy, and debate focus on two functions of assessment and evaluation. *Assessment for learning* is the use of diagnostic and formative assessment to improve student achievement. This is usually the concern of individual classroom teachers or groups of teachers. *School and district improvement* is the use of assessment results to stimulate local improvement initiatives, guide decision making, and monitor results. Often referred to as "data-driven decision making," it aims to demonstrate accountability and improve student learning. This process is often led by school or district administrators and may be mandated by a level of government.

In recent years, research on assessment and evaluation has accelerated as policy-makers recognize what teachers have long known—assessment has a powerful effect on what is taught, how it is taught, and what and how well students learn. This renewed interest in assessment places complex demands on teachers. To a large extent, the success of students and entire systems depends on collecting and interpreting valid assessment data that are aligned with the curriculum and offer an accurate view of student achievement.

An analysis of more than 250 research studies worldwide indicates that effective assessment for learning can improve student achievement substantially, particularly for low achievers. The results suggest that effective assessment could be one of the most powerful intervention techniques ever identified. In Canada, for example, changing teacher practice to incorporate principles of effective assessment could improve average student achievement by one to two levels or letter grades, or move Canada from the middle to the top tier of countries in international mathematics studies such as TIMSS.[2]

Rick Stiggins, founder of the Assessment Training Institute in Portland, Oregon, suggests that quality classroom assessment depends on a variety of factors, including aligning assessment to curriculum and standards and developing precise assessment tools that are capable of accurately describing the learning in question.[3] He argues that student achievement is most improved when learning and assessment goals are transparent, and when students receive descriptive feedback instead of just a mark. Such substantive feedback directs students' own learning efforts by

- communicating what students have or have not achieved (rather than comparisons to others), including improvements to earlier work;
- explaining, in a timely fashion, whether students are right or wrong and why;
- outlining specific ways in which students can improve the quality of their work; and
- inviting students to suggest how they can strengthen their skills and improve their own performance.[4]

Assessment for learning must also develop students' self-assessment skills. Given the opportunity, a constructive and supportive classroom environment, and explicit strategies and tools, students can set learning goals that enhance their development and foster independence, responsibility, and self-esteem.

Assessment is not an event, but a continuous process embedded in instruction. National efforts have been directed at helping teachers select and design assessment tools that maximize the feedback on student progress but are not disruptive to teaching and learning. This has been accomplished by adding new tools to the assessment repertoire of the classroom teacher: mathematics journals and portfolios, reports on investigations, observation notes, open-ended rich assessment tasks, rubrics, student conferencing, and exemplars.

Assessment is not an event, but a continuous process embedded in instruction.

Regular interpretation of students' work has immediate implications for classroom instruction, including the kinds of questions asked, tasks posed, and homework assigned. Studies have shown that when teachers learn to interpret specific information communicated in their students' work, they make judicious use of that information to shape their instruction in a highly effective manner.

From Assessment to Evaluation

Formal and informal assessment provides teachers with a great deal of information about what students know. Assessment information also offers insight into students' attitudes towards mathematics and beliefs about themselves as "doers" of mathematics.

But how do teachers *evaluate* this record of information gathered from problems, tasks, quizzes, presentations, and projects? Evaluation requires teachers to use the assessment evidence they have gathered over time and compare it to a set of established standards. Evaluation is challenging because it must determine how the student measures up to an objective standard. This process is accomplished best when using a generous amount of assessment information or by analyzing assessment data that represent a whole range of thinking skills, such as problem solving, reasoning, communication, and conceptual understanding.

Evaluation is challenging because it must determine how the student measures up to an objective standard.

The final step in the evaluation process involves generating a mark or grade for the student based on the assessment data. The mark can be a letter grade or a numerical percentage. Teachers generate this final grade in order to communicate their judgment about the student to the broader community, including other teachers, administrators, and parents. Most teachers approach grading systematically by assigning different weights to different results and finding some valid "average" to represent the student's performance over time. However, many teachers feel restricted by reducing all of their assessment data to a single number or letter because they are gathering such a broad range of information about their students' mathematical achievement.

What Are the Implications?

- Assessment tools must align with the curriculum by stating learning outcomes/expectations.
- Assessment tasks must reveal what students know and what they can do, i.e., *how* they solve problems, not just the solutions they reach.

- Assessment tasks must stimulate mathematics learning.
- Being familiar with the assessment tools and marking schemes helps students understand how they are being assessed and evaluated.
- Teachers require information on using assessment information to guide and/or adjust their classroom instruction.
- Assignments and tests must be weighted based on the type of thinking required, e.g., higher-order thinking tasks that demand problem-solving, reasoning, communication, or conceptual understanding must be given more weight than lower-order tasks that emphasize computational skills.

Assessment tools must align with the curriculum by stating learning outcomes/ expectations.

Endnotes

1. For more information about the Assessment Reform Group, visit their Web site at <www.assessment-reform-group.org.uk>.
2. P. Black & D. Wiliam (1998), Assessment and classroom learning, *Assessment in Education, 5*(1), pp. 7–74.
3. R. Stiggins (2001), Making classroom assessment instructionally relevant. Paper presented at the annual meeting of the American Educational Research Association, Seattle, 2001.
4. R.L. Bangert-Downs, C.C. Kulik, J.A. Kulik, and M. Morgan (1991), The instructional effects of feedback in test-like events, *Review of Educational Research, 61*(2), 213–238; Terry Crooks (2001), The validity of formative assessments—paper prepared for the annual meeting of the British Educational Research Association, Leeds England, 13–15 September; P. Tunstall and C. Gipps (1996), How does your teacher help you to make your work better? Children's understanding of formative assessment, *The Curriculum Journal*, *3*(7), 185–203.

For Further Reading

Marzano, R.J., Pickering, D.J., & Pollock, J.E. (2001). *Classroom instruction that works: Research-based strategies for increasing student achievement.* Alexandria, VA: ASCD.

Equity

What Research Tells Us

Equity is one of the NCTM guiding principles and a major consideration for Canadian teachers working with diverse populations. Equity involves recognizing differences in background, gender, learning styles, language, and physical challenges. The equity principle states that educators have an obligation to help every student become successful in mathematics and that high expectations are appropriate for everyone.

Background Children arrive at school with differences in number experiences and conceptual understanding. Remedial interventions, if required, must start early—as soon as teachers diagnose specific difficulties. Early remediation can "level the playing field" and nurture success. Strong evidence suggests that a three-year difference in number knowledge in the early years of school can become a seven-year difference in low-achieving students after the first ten years of school.[1]

Gender There is no evidence that girls are intellectually less capable than boys when *doing* mathematics.[2] However, girls and boys may differ in their *attitudes* towards mathematics. The data show that elementary girls outperform boys in mathematics achievement up to about Grade 6. In spite of their better performance, they are less confident about their ability to do mathematics and more likely to believe that mathematics is a hard subject. Boys believe that they are good at mathematics even if their scores are lower than girls' scores.[3] As girls progress through the grades, they report more doubt about their ability to succeed in mathematics. This belief results in diminished classroom participation and "dropping out" of mathematics.[4]

Girls may believe that mathematics is not essential to their future careers. By Grade 3, many girls have already decided that mathematics is irrelevant.[5]

Learning Styles All children can learn, but not in the same way. *Visual* learners need to see the teacher's body language and facial expression to fully understand the content of a lesson. They may think in pictures and learn best from visual resources such as illustrated textbooks. *Auditory* learners prefer verbal lectures and discussions. They learn best by talking things through and listening to what others have to say. *Tactile/kinesthetic* learners require a hands-on approach, which allows them to actively explore the physical world around them.

Language The NCTM cautions that ESL students might require special attention in order to participate meaningfully in class discussions. They might also need accommodation in assessment to ensure that their *mathematics proficiency* is being evaluated, not their facility with the English language.

Physical Ability The NCTM states that students with physical challenges must have the flexibility to learn and be assessed using preferred modes of communication, such as oral versus written. They may also require additional time to complete assignments. Technology, too, can benefit disabled students by helping them overcoming certain physical barriers.

Technology can benefit disabled students by helping them overcoming certain physical barriers.

What Are the Implications?

- Educators must commit to the principle of "equal outcomes/ expectations" for all students of mathematics.
- Rich early-childhood mathematics experiences should be valued and made a social priority.
- Resources must provide multiple approaches to learning so that all children are valued and accommodated.

Endnotes

1. R.J. Wright et al. (2000), *Early numeracy: Assessment for teaching and intervention*, London: Sage Publications; D.C. Geary (1996), *Children's mathematical development: Research and practical applications*, Washington: American Psychological Association.
2. Education Development Center (1995), *Equity in education series: Gender-fair math*, Newton, MA.
3. EQAO, *Ontario provincial report on achievement, 2000–2001.*
4. R. Tobias (1992), *Nurturing at-risk youth in math and science: Curriculum and teaching considerations*, Bloomington, IN: National Education Service.
5. N. Burton (1995), Trends in mathematics achievement for young men and women, *Prospects for school mathematics*, I.M. Carl (Ed.), Reston, VA: National Council of Teachers of Mathematics, pp. 115–130.

Technology, used *purposefully* in the classroom, can enrich the mathematical learning experiences of students.

Technology

What Research Tells Us

Although calculators and computers have made significant contributions to the world of mathematics, many teachers, parents, and administrators still question their usefulness in the classroom. However, technology, used *purposefully* in the classroom, can enrich the mathematical learning experiences of students.[1] Technological tools—from calculators to spreadsheets to dynamic geometry tool kits—enhance learning by allowing students to explore and develop concepts, apply and extend problem-solving skills, and increase efficiency. Canadian curriculum documents endorse the use of specific technologies at various points within their programs of study.

The NCTM states the following:

> Students can learn more mathematics more deeply with the appropriate use of technology.… Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions. In mathematics instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics.[2]

Principles and Standards notes that technologies can support learning and teaching in the following ways:

- By allowing for efficient computation, by organizing data, or by providing a visual representation of a mathematical idea, technological tools free students to focus on conjecturing, decision making, and problem solving.
- Technology allows children at the primary level to perform tasks that were previously inaccessible, such as investigating the characteristics of geometric shapes or computing with large numbers.
- Technological tools provide instantaneous feedback. For example, when students manipulate data in a spreadsheet program, they quickly understand how the different cells of data are interdependent and how modifying selected values forces dependent values to change.

- Using technology, teachers can select a wider range of mathematical tasks, especially in those areas where technology provides efficiency, such as graphing or computing.[3]

By supplying immediate feedback, technological tools also give children the opportunity to work on process skills such as conjecturing and hypothesis formulation. In addition, because technology allows for repetition, it supports both individual learning and metacognition. Students can look back to see how a task was successfully achieved so that the process can be replicated or adapted in the future.

Technology allows children at the primary level to perform tasks that were previously inaccessible, such as investigating the characteristics of geometric shapes or computing with large numbers.

What Are the Implications?

- Students benefit from integrating technology and using it purposefully in mathematics learning.
- Technology should be viewed as supporting learning, not replacing it.
- Opportunities to use technology—the how, why, and when—must be made explicit in learning resources.
- Exemplars for using technology purposefully in the classroom support student learning.

Endnotes

1. M.E. Brenner et al. (1997), Learning by understanding: The role of multiple representations in learning algebra, *American Educational Research Journal, 34*(4), 663–689; D.H. Clements (1999), Effective use of computers with young children, *Mathematics in the early years*, J.V. Copley (Ed.), Reston, VA: National Council of Teachers of Mathematics, pp. 119–128; M.K. Heid (1997), The technological revolution and the reform of mathematics, *American Journal of Education, 106*(1), 5–61; J.H. Sandholtz et al. (1997), Teaching with technology: Creating student-centered classrooms, New York: Teachers College Press.
2. National Council of Teachers of Mathematics (2000), *Principles and standards for school mathematics*, p. 25.
3. Ibid., pp. 24–27.

Teacher Knowledge

> When a teacher doesn't have a deep understanding and affinity for mathematics or science ... it's hard to imagine how that teacher will help students understand and get excited by these subjects.[1]
>
> — Lee Shulman, president, Carnegie Foundation for the Advancement of Teaching

What Research Tells Us

The NCTM and the National Research Council have noted that effective teachers of mathematics require several varieties of "mathematical knowledge." One type of knowledge is the discipline of mathematics itself—its facts, procedures, and conceptual framework (both the concepts and how the concepts are connected). Research undertaken by Pearson Canada has shown that teachers are asking for more support in the areas of mathematics content, developmental progression in mathematics, the language of mathematics, and problem solving.

In order to instruct mathematics effectively, teachers require more than the ability to teach facts and procedures; they require an understanding of the conceptual foundation of mathematics. They also need to have the ability to explain and clarify mathematical ideas (e.g., the underlying properties for adding and multiplying) and to problem-solve in a variety of ways.[2]

Closely related to this knowledge domain are two issues—teacher understanding of how mathematical ideas can best be represented in the classroom and teacher beliefs about mathematics. In the classroom, students learn abstract ideas by identifying their underlying concepts. Research has shown that teachers with a strong conceptual understanding of mathematics usually succeed in helping students develop their own understanding of mathematical ideas. Other studies indicate that conceptual understanding allows teachers to use students' solutions, explanations, and questions to reveal and clarify mathematical concepts.[3]

Confident teachers of mathematics allow students' solutions to emerge and be used as a springboard for further discussion. Teaching mathematics successfully requires more than knowing the discipline; it requires knowing how children learn mathematics developmentally and how to create instructional situations that replicate those developmental stages.[4]

Teacher beliefs about mathematics are also relevant. If teachers believe that mathematics is static, linear, and rule oriented, they will teach mathematics in that fashion—by emphasizing rules and memorization. On the other hand, when teachers believe that mathematics is dynamic and ever-changing, they structure their classrooms in a way that underscores that belief. Research has shown that teacher beliefs about mathematics affect not only the way they teach but also their choice of activities. This relationship between belief and practice exists in either a teacher- or child-centred classroom.[5]

Teaching mathematics successfully requires more than knowing the discipline; it requires knowing how children learn mathematics developmentally and how to create instructional situations that replicate those developmental stages.

What Are the Implications?

- Teachers benefit from professional-development training opportunities to increase their understanding of mathematics content.
- Mathematics programs for use in the classroom must provide clear direction for teachers, not only on *what* to teach (curriculum) but also *how* to teach (methodology).
- Resources must include explicit instructional strategies to support teaching through problem solving and communication.

Endnotes

1. Appreciating good teaching—a conversation with Lee Shulman (2001), *Educational Leadership, 58*(5). To see the full interview, go to <www.ascd.org/readingroom/edlead/0102/tell.html>.
2. National Research Council (2001), *Adding it up: Helping children learn mathematics*, J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, Washington, DC: National Academy Press, pp. 371–373.
3. Ibid., p. 378. See footnotes 20 to 23 for research.
4. J. Stigler & J. Hiebert (1999), *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*, New York: Free Press, p. 88.
5. S. Raizen, T. Britton, et al. (1996), *Bold ventures: Case studies of US innovations in science and mathematics education, vol. 3.,* Dordrecht: Kluwer; P. Black & J. Myron Atkin (Eds.) (1996), *Changing the subject: Innovations in science, mathematics and technology education*, London and New York: Routledge.

For Further Reading

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Professional Development

What Research Tells Us

Across Canada, curriculum reform has prompted a re-examination of the role of professional development. On-going, high-quality professional development can drive, support, and sustain long-term change.

In the area of mathematics, professional development can enrich teacher knowledge of content, deepen pedagogical understanding, and broaden the teacher's repertoire of effective instructional and assessment practices. The NCTM has recommended professional development opportunities that enhance the knowledge and skills required to implement the curriculum, expand problem-solving ability, overcome barriers to learning in the classroom, and devise action plans to meet specific teaching needs.[1]

> Professional development can no longer be viewed as an event that occurs on a particular day of the school year; rather, it must become part of the daily work life of educators.
>
> — Cathy J. Cook and Carol Fine

In their 1989 analysis of professional development, Dennis Sparks and Susan Loucks-Horsey identified several characteristics of effective programs:

- Activities take place in school settings.
- Teachers are actively and collaboratively involved in planning, setting goals, and selecting activities.
- Self-instruction is emphasized, and teachers can select from a variety of "differentiated training opportunities."
- Ongoing support and resources are provided.
- Training is concrete and includes ongoing feedback, supervised trials, and assistance on request.[2]

A 1996 survey of various models identified additional characteristics of effective professional development for mathematics and science teachers:

- Programs are driven by a clear vision of effective classroom practice.
- Teachers are provided with learning experiences that enhance their understanding of major discipline concepts.
- Teachers have opportunities for instruction that mirror the methods to be used with students.[3]

Deborah Schifter provides a unique perspective on professional development. Her reflective teaching model recognizes the classroom teacher as having the most important research role: teachers identify problems, search for possible causes and solutions, test solutions in practice, validate their observations, and discuss their findings with colleagues. As researchers "in the field," teachers often emerge with a new appreciation for the complexity of learning and teaching.[4]

In a national survey conducted by Pearson Canada, 88 percent of teacher respondents and 91 percent of principal respondents ranked learning opportunities as a "very important" professional need.

What Are the Implications?

- Drawing on different models of professional development helps educators create successful local programs.
- Teachers require the opportunity to collaborate actively at the school level—to become a community of learners.
- Professional development must be sustained and connected over time.
- Teachers benefit from doing mathematics through the eyes of their students—in the same setting and using the same resources and materials.
- Professional development opportunities must provide a venue for teachers to reflect on and evaluate their classroom experiences.

The most effective professional development model is thought to involve follow-up activities, usually in the form of long-term support, coaching in teachers' classrooms, or ongoing interaction with colleagues.[5]

— Deborah Ball

Endnotes

1. For the complete NCTM Academy Mission Statement, go to <www.nctm.org/academy/mission.htm>.
2. D. Sparks & S. Loucks-Horsley (1989), Five models of staff development for teachers, *Journal of Staff Development, 10*(4), 40–55.
3. For the complete summary of important principles, see S. Loucks-Horsey et al. (1996), Professional development for mathematics and science education: A synthesis of standards, *National Institute for Science Education Brief, 1*(1). Go to <www.wcer.wisc.edu/nise/Publications/> and click on Briefs.
4. D. Schifter (1995), *What's happening in math class, volumes 1 and 2*, New York: Teachers College Press, 1995.
5. D. Ball (1996), Teacher learning and the mathematics reforms: What we think we know and what we need to learn, *Phi Delta Kappan 78*(7).

Program Manageability

Because teachers are being asked to change the way they approach mathematics in the classroom, new resources must provide "support, direction, and explicit instructional strategies."

What Research Tells Us

Curriculum reform in Canada has driven change in almost every area of education. Because the current mathematics curriculum is comprehensive and rigorous, teachers require a manageable program of instruction. Not only should resources provide a 100 percent match to the curriculum (see Meeting the Curriculum, page 24), they should also be concise and easy to use, providing teacher support in several key areas. In teacher interviews conducted by Pearson Canada, respondents stated that they would like programs and resources to accomplish the following:

- *Anticipate the challenges of implementing the curriculum.* Teachers recognize that curriculum reform represents a shift from teaching information to creating rich learning opportunities for mastering a discipline. Because teachers are being asked to change the way they approach mathematics in the classroom, new resources must provide "support, direction, and explicit instructional strategies."
- *Link to the curriculum and enable teachers to approach mathematics concepts in sequence.* Teachers say that "a disjointed scope and sequence" does not help to build conceptual understanding in students.
- *Provide background information on mathematical concepts.* Teachers want the mathematics "why" to be included in new resources. Scaffolding for teachers as well as students is viewed as a need.
- *Meet the needs of a wide variety of learners.* Students with different cognitive strengths require equal opportunities to learn. Teachers also require strategies for ESL students, who may find the new language-based approach to mathematics difficult.
- *Address the issue of split grades.* It is estimated that 20 percent of Canadian children are in a split-grade class.[1] Teaching two sets of outcomes in the time designed for teaching one grade poses several challenges.[2]

What Are the Implications?

- The program must offer direction and instructional strategies, link to the curriculum, and provide a model sequence of the mathematics ideas to be taught.
- The program should offer practical strategies to deal with assessment and reporting, instruction, special needs, and split grades.
- Resources should enable teachers to apply professional judgment to meet individual student needs.

Endnotes

1. H. Dolik (March 2002), High expectations: The challenge of the modern multigrade classroom, *Professionally Speaking*. To see the entire article, go to <www.oct.on.ca/english/ps/march_2002/high_expectations.asp>.
2. Ibid. See also L. Kuen (1993), Changing teaching practice: Teachers' aspirations meet school realities, *BCTF Research Report*, No. 1. To see this report, visit <www.bctf.bc.ca/ResearchReports/93ei05>.

Parents' Involvement

What Research Tells Us

Many parents believe that the new "new math" being taught today is dramatically different from the mathematics they were taught in school. In fact, there are similarities and differences. In their own schooling, most adults experienced a mathematics curriculum that emphasized facts and procedures for computing. By contrast, today's curriculum maintains a careful balance between developing conceptual understanding and computational fluency. There is a strong perception that parents require more information about the mathematics their children are doing. Parents can be assured that today's mathematics curriculum enables their children to

- know and apply basic mathematics skills
- solve problems using many different strategies
- think independently
- reason skillfully in diverse situations
- communicate effectively regarding solutions to problems and methods for solving them
- work alone and in groups to solve problems

All Canadian curriculum documents note that there are three partners in the learning process—students, teachers, and parents.

There is no doubt that parent involvement supports student success. All Canadian curriculum documents note that there are three partners in the learning process—students, teachers, and parents. The importance of parent involvement is reflected in educational policies and practices throughout Canada and is actively supported at all levels—national, provincial, and local. One such initiative is *Working Together in Mathematics Education*, a handbook for parents created by Alberta Learning.[1] This handbook provides an overview of the new mathematics curriculum and offers suggestions for supporting student learning at home.

One topic of great interest to parents is homework. Examining the link between student achievement and homework, social psychologist Harris Cooper found that the results differed for elementary, junior-high, and high-school students.[2] Although he could find no direct link between achievement and homework with younger children, he noticed a strong relationship between homework and the achievement of high-school students. In addition, he found that all students benefit from developing good study habits and a positive attitude about learning—for example, believing that learning can take place both outside and inside school.

Cooper suggests the following guidelines for homework policies:

- Districts, schools, and classrooms should coordinate their homework policies and communicate them to parents.
- The amount and type of homework should be based on students' developmental level and the degree of support at home. Most educators are guided by the 10-minute rule, that is, 10 minutes per grade level per night for all subjects.
- Homework can be effective by serving different purposes at different grade levels. Younger students can also be taught to recognize learning in everyday and extra-curricular activities.

Pearson surveys revealed that parents want to be more involved with their children's mathematical learning at home. When asked how they would provide support, the majority of respondents favoured homework that incorporates practice, examples, and real-life situations.

What Are the Implications?

- Schools should create opportunities to explain the curriculum and mathematics materials to parents.
- Students benefit when schools encourage parent involvement, communicate with parents regularly, and support learning at home.
- Time spent for homework should be age appropriate and coordinated with other subjects. A reasonable guideline is 10 minutes per grade.

Endnotes

1. Alberta Education, Curriculum Standards Branch (1996), *Working together in mathematics education*. Visit <http://ednet.edc.gov.ab.ca>.
2. For a summary of Cooper's findings, see H. Ross (April 2001), Homework for all—in moderation, *Educational Leadership, 58*(7).